## 6678 <br> Edexcel GCE

## Mechanics M2

(New Syllabus)

## Advanced/Advanced Subsidiary <br> Thursday 21 June 2001 - Afternoon <br> Time: 1 hour 30 minutes

Materials required for examination
Answer Book (AB16)
Graph Paper (ASG2)
Mathematical Formulae (Lilac)

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

## Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of $g$ is required, take $g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$
When a calculator is used, the answer should be given to an appropriate degree of accuracy.
Information for Candidates
A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions
This paper has seven questions.

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answer without working may gain no credit.

1. At time $t$ seconds, a particle $P$ has position vector $r$ metres relative to fixed origin $O$, where

$$
\mathbf{r}=\left(t^{2}+2 t\right) \mathbf{i}+\left(t-2 t^{2}\right) \mathbf{j}
$$

Show that the acceleration of $P$ is constant and find its magnitude.

Figure 1


Figure 1 shows a decoration which is made by cutting 2 circular discs from sheet of uniform card. The discs are joined so that they touch at a point $D$ on the circumference of both discs. The discs are coplanar and have centres $A$ and $B$ with radii 10 cm and 20 cm respectively.
(a) Find the distance of the centre of mass of the decoration from B.

The point $C$ lies on the circumference of the smaller disc and $\angle C A B$ is a right angle. The decoration is freely suspended from C and hangs in equilibrium.
(b) Find, in degrees to one decimal place, the angle between AB and the vertical.
3. A uniform ladder $A B$, of mass $m$ and length $2 a$, has one end $A$ on rough horizontal ground. The coefficient of friction between the ladder and the ground horizontal ground. The coefficient of friction between the ladder and the ground
is 0.5 . The other end $B$ of the ladder rests against a smooth vertical wall. The is 0.5 . The other end $B$ of the ladder rests against a smooth vertical wall. The
ladder rests in equilibrium in a vertical plane perpendicular to the wall, and ladder rests in equilibrium in a vertical plane perpendicular to the wall, and
makes an angle of $30^{\circ}$ with the wall. A man of mass 5 m stands on the ladder makes an angle of $30^{\circ}$ with the wall. A man of mass 5 m stands on the ladder
which remains in equilibrium. The ladder is modelled as a uniform rod and the man as a particle. The greatest possible distance of the man from $A$ is $k a$.

Find the value of $k$.
4. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ lie in a vertical plane, $\mathbf{i}$ being horizontal and $\mathbf{j}$ vertical. A ball of mass 0.1 kg is hit by a bat which gives it an impulse of $(3.5 \mathbf{i}+3 \mathbf{j}) \mathrm{Ns}$. The velocity of the ball immediately after being hit is $(10 \mathbf{i}+25 \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}$.
(a) Find the velocity of the ball immediately before it is hit.

In the subsequent motion the ball is modelled as a particle moving freely under gravity. When it is hit the ball is 1 m above horizontal ground.
(b) Find the greatest height of the ball above the ground in the subsequent motion.

The ball is caught when it is again 1 m above the ground.
(c) Find the distance from the point where the ball is hit to the point where it is caught.
5. A child is playing with a small model of a fire-engine of mass 0.5 kg and a straight, rigid plank. The plank is inclined at an angle $\alpha$ to the horizontal. The fire-engine is projected up the plank along a line of greatest slope. The non-gravitational resistance to the motion of the fire-engine is constant and has magnitude $R$ newtons.

When $\alpha=20^{\circ}$ the fire-engine is projected with an initial speed of $5 \mathrm{~m} \mathrm{~s}^{-1}$ and first comes to rest after travelling 2 m .
(a) Find, to 3 significant figures, the value of $R$.

When $\alpha=40^{\circ}$ the fire-engine is again projected with an initial speed of $5 \mathrm{~m} \mathrm{~s}^{-1}$.
(b) Find how far the fire-engine travels before first coming to rest.
$\qquad$
6. A particle $A$ of mass $2 m$ is moving with speed $2 u$ on a smooth horizontal table. The particle collides directly with a particle $B$ of mass $4 m$ moving with speed $u$ in the same direction as $A$. The coefficient of restitution between $A$ and $B$ is $\frac{1}{2}$.
(a) Show that the speed of $B$ after the collision is $\frac{3}{2} u$.
(b) Find the speed of $A$ after the collision.

Subsequently $B$ collides directly with a particle $C$ of mass $m$ which is at rest on the table. The coefficient of restitution between $B$ and $C$ is $e$. Given that there are no further collisions,
(c) find the range of possible values for $e$.
7.


At time $t=0$ a small package is projected from a point $B$ which is 2.4 m above a point $A$ on horizontal ground. The package is projected with speed $23.75 \mathrm{~m} \mathrm{~s}^{-1}$ at an angle $\alpha$ to the horizontal, where $\tan \alpha=\frac{4}{3}$. The package strikes the ground at the point $C$, as shown in Fig. 2. The package is modelled as a particle moving freely under gravity.
(a) Find the time taken for the package to reach $C$.

A lorry moves along the line $A C$, approaching $A$ with constant speed $18 \mathrm{~m} \mathrm{~s}^{-1}$. At A lorry moves along the line $A C$, approaching $A$ with constant speed $18 \mathrm{~m} \mathrm{~s}^{-1}$. At
time $t=0$ the rear of the lorry passes $A$ and the lorry starts to slow down. It comes to rest $T$ seconds later. The acceleration, $a \mathrm{~m} \mathrm{~s}^{-2}$ of the lorry at time $t$ seconds is given by

$$
a=-\frac{1}{4} t^{2}, \quad 0 \leq t \leq T .
$$

(b) Find the speed of the lorry at time $t$ seconds.
(c) Hence show that $T=6$.
(d) Show that when the package reaches $C$ it is just under 10 m behind the rear of the moving lorry.

## 6678 <br> Edexcel GCE

## Mechanics M2 <br> (New Syllabus) <br> Advanced/Advanced Subsidiary <br> Friday 25 January 2002 - Morning <br> Time: $\mathbf{1}$ hour 30 minutes

Materials required for examination
Answer Book (AB16)
Graph Paper (ASG2)
Mathematical Formulae (Lilac)

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculator such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G

## Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of $g$ is required, take $g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions
This paper has seven questions. Pages 7 and 8 are blank.

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. A particle of mass 4 kg is moving in a straight horizontal line. There is a constant resistive force of magnitude $R$ newtons. The speed of the particle is reduced from $25 \mathrm{~m} \mathrm{~s}^{-1}$ to rest over a distance of 200 m .

Use the work-energy principle to calculate the value of $R$.
2. A van of mass 1500 kg is driving up a straight road inclined at an angle $\alpha$ to the horizontal, where $\sin \alpha=\frac{1}{12}$. The resistance to motion due to non-gravitational forces is modelled as a constant force of magnitude 1000 N .

Given that initially the speed of the van is $30 \mathrm{~m} \mathrm{~s}^{-1}$ and that the van's engine is working at a rate of 60 kW ,
(a) calculate the magnitude of the initial decleration of the van.

When travelling up the same hill, the rate of working of the van's engine is increased to 80 kW . Using the same model for the resistance due to nongravitational forces,
(b) calculate in $\mathrm{m} \mathrm{s}^{-1}$ the constant speed which can be sustained by the van at this rate of working.
(c) Give one reason why the use of this model for resistance may mean that your answer to part (b) is too high.
3. A particle $P$ of mass 0.3 kg is moving under the action of a single force F newtons. At time $t$ seconds the velocity of $P, \mathrm{v} \mathrm{m} \mathrm{s}^{-1}$, is given by

$$
\mathbf{v}=3 t^{2} \mathbf{i}+(6 t-4) \mathbf{j} .
$$

(a) Calculate, to 3 significant figures, the magnitude of $\mathbf{F}$ when $t=2$.

When $t=0, P$ is at the point $A$. The position vector of $A$ with respect to a fixed origin $O$ is $(3 \mathbf{i}-4 \mathbf{j}) \mathrm{m}$. When $t=4, P$ is at the point $B$.
(b) Find the position vector of $B$.
4.


Figure 1 shows a template made by removing a square $W X Y Z$ from a uniform triangular lamina $A B C$. The lamina is isosceles with $C A=C B$ and $A B=12 a$. The mid-point of $A B$ is $N$ and $N C=8 a$. The centre $O$ of the square lies on $N C$ and $O N=2 a$. The sides $W X$ and $Z Y$ are parallel to $A B$ and $W Z=2 a$. The centre of mass of the template is at $G$.
(a) Show that $N G=\frac{30}{11} a$.

The template has mass $M$. A small metal stud of mass $k M$ is attached to th template at $C$. The centre of mass of the combined template and stud lies on YZ By modelling the stud as a particle,
(b) calculate the value of $k$
5.

## Figure 2



Figure 2 shows a horizontal uniform pole $A B$, of weight $W$ and length $2 a$. The end $A$ of the pole rests against a rough vertical wall. One end of a light inextensible string $B D$ is attached to the pole at $B$ and the other end is attached to the wall at $D$. A particle of weight $2 W$ is attached to the pole at $C$, where $B C=x$. the wall at $D$. A particle of weight $2 W$ is attached to the pole at $C$, where $B C=x$.
The pole is in equilibrium in a vertical plane perpendicular to the wall. The string $B D$ is inclined at an angle $\theta$ to the horizontal, where $\sin \theta=\frac{3}{5}$. The pole is modelled as a uniform rod.
(a) Show that the tension in $B D$ is $\frac{5(5 a-2 x)}{6 a} W$.

The vertical component of the force exerted by the wall on the pole is $\frac{7}{6} \mathrm{~W}$. Find
(b) $x$ in terms of $a$,
(c) the horizontal component, in terms of $W$, of the force exerted by the wall on the pole.
6. A smooth sphere $P$ of mass $m$ is moving in a straight line with speed $u$ on a smooth horizontal table. Another smooth sphere $Q$ of mass $2 m$ is at rest on the table. The sphere $P$ collides directly with $Q$. After the collision the direction of motion of $P$ is unchanged. The spheres have the same radii and the coefficient of restitution between $P$ and $Q$ is $e$. By modelling the spheres as particles,
(a) show that the speed of $Q$ immediately after the collision is $\frac{1}{3}(1+e) u$,
(b) find the range of possible values of $e$.

Given that $e=\frac{1}{4}$,
(c) find the loss of kinetic energy in the collision.
(d) Give one possible form of energy into which the lost kinetic energy has been transformed.

TURN OVER FOR QUESTION 7
7.


A rocket $R$ of mass 100 kg is projected from a point $A$ with speed $80 \mathrm{~m} \mathrm{~s}^{-1}$ at an angle of elevation of $60^{\circ}$, as shown in Fig. 3. The point $A$ is 20 m vertically above a point $O$ which is on horizontal ground. The rocket $R$ moves freely under gravity. At $B$ the velocity of $R$ is horizontal. By modelling $R$ as a particle, find
(a) the height in $m$ of $B$ above the ground,
(b) the time taken for $R$ to reach $B$ from $A$.

When $R$ is at $B$, there is an internal explosion and $R$ breaks into two parts $P$ and $Q$ of masses 60 kg and 40 kg respectively. Immediately after the explosion the velocity of $P$ is $80 \mathrm{~m} \mathrm{~s}^{-1}$ horizontally away from $A$. After the explosion the paths of $P$ and $Q$ remain in the plane $O A B$. Part $Q$ strikes the ground at $C$. By modelling $P$ and $Q$ as particles,
(c) show that the speed of $Q$ immediately after the explosion is $20 \mathrm{~m} \mathrm{~s}^{-1}$,
(d) find the distance $O C$.

## 6678

## Edexcel GCE

## Mechanics M2

## Advanced/Advanced Subsidiary <br> Tuesday 18 June 2002 - Afternoon <br> Time: 1 hour 30 minutes

Materials required for examination
Answer Book (AB16)
Mathematical Formulae (Lilac)
Graph Paper (ASG2)

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculator 48 G

## Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of $g$ is required, take $g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## nformation for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions
This paper has seven questions. Pages 7 and 8 are balnk.

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. The velocity $\mathbf{v} \mathrm{m} \mathrm{s}^{-1}$ of a particle $P$ at time $t$ seconds is given by

$$
\mathbf{v}=(3 t-2) \mathbf{i}-5 t \mathbf{j} .
$$

(a) Show that the acceleration of $P$ is constant.

At $t=0$, the position vector of $P$ relative to a fixed origin $O$ is $3 \mathbf{i} \mathrm{~m}$.
(b) Find the distance of $P$ from $O$ when $t=2$.
2. A particle $P$ moves in a straight line so that, at time $t$ seconds, its acceleration $a \mathrm{~m} \mathrm{~s}^{-2}$ is given by

$$
a= \begin{cases}4 t-t^{2}, & 0 \leq t \leq 3, \\ \frac{27}{t^{2}}, & t>3\end{cases}
$$

At $t=0, P$ is at rest. Find the speed of $P$ when
(a) $t=3$,
(b) $t=6$.
3.

Figure 1


Figure 1 shows the path taken by a cyclist in travelling on a section of a road. When the cyclist comes to the point $A$ on the top of a hill, she is travelling at $8 \mathrm{~m} \mathrm{~s}^{-1}$. She descends a vertical distance of 20 m to the bottom of the hill. The road then rises to the point $B$ through a vertical distance of 12 m . When she reaches $B$, her speed is $5 \mathrm{~m} \mathrm{~s}^{-1}$. The total mass of the cyclist and the cycle is 80 kg and the total distan 5 m s . The total long the road from $A$ to $B$ is 500 m . By modelling the resistance to the motion of the cyclist as of constant magnitude 20 N ,
(a) find the work done by the cyclist in moving from $A$ to $B$.

At $B$ the road is horizontal. Given that at $B$ the cyclist is accelerating at $0.5 \mathrm{~m} \mathrm{~s}^{-2}$,
(b) find the power generated by the cyclist at $B$.
4.

Figure 1


A uniform lamina $L$ is formed by taking a uniform square sheet of material $A B C D$, of side 10 cm , and removing the semi-circle with diameter $A B$ from the square, as shown in Fig. 2.
(a) Find, in cm to 2 decimal places, the distance of the centre of mass of the lamina $L$ from the mid-point of $A B$.
[The centre of mass of a uniform semi-circular lamina, radius $a$, is at a distance $\frac{4 a}{3 \pi}$ from the centre of the bounding diameter.]

The lamina is freely suspended from $D$ and hangs at rest.
(b) Find, in degrees to one decimal place, the angle between $C D$ and the vertical.
5. A particle is projected from a point with speed $u$ at an angle of elevation $\alpha$ above the horizontal and moves freely under gravity. When it has moved a horizontal distance $x$, its height above the point of projection is $y$.
(a) Show that

$$
\begin{equation*}
y=x \tan \alpha-\frac{g x^{2}}{2 u^{2}}\left(1+\tan ^{2} \alpha\right) \tag{5}
\end{equation*}
$$

A shot-putter puts a shot from a point $A$ at a height of 2 m above horizontal ground. The shot is projected at an angle of elevation of $45^{\circ}$ with a speed of $14 \mathrm{~m} \mathrm{~s}^{-1}$. By modelling the shot as a particle moving freely under gravity,
(b) find, to 3 significant figures, the horizontal distance of the shot from $A$ when the shot hits the ground,
(c) find, to 2 significant figures, the time taken by the shot in moving from $A$ to reach the ground.
6. A small smooth ball $A$ of mass $m$ is moving on a horizontal table with speed $u$ when it collides directly with another small smooth ball $B$ of mass 3 m which is at rest on the table. The balls have the same radius and the coefficient of restitution between the balls is $e$. The direction of motion of $A$ is reversed as a result of the collision.
(a) Find, in terms of $e$ and $u$. the speeds of $A$ and $B$ immediately after the collision.

In the subsequent motion $B$ strikes a vertical wall, which is perpendicular to the direction of motion of $B$, and rebounds. The coefficient of restitution between $B$ and the wall is $\frac{3}{4}$.

Given that there is a second collision between $A$ and $B$,
(b) find the range of values of $e$ for which the motion described is possible.
7.

Figure 3


A straight $\log A B$ has weight $W$ and length $2 a$. A cable is attached to one end $B$ of the log. The cable lifts the end $B$ off the ground. The end $A$ remains in contact with the log. The cable lifts the end $B$ off the ground. The end $A$ remains in contact with the
ground, which is rough and horizontal. The $\log$ is in limiting equilibrium. The log makes an angle $\alpha$ to the horizontal, where $\tan \alpha=\frac{5}{12}$. The cable makes an angle $\beta$ to the horizontal, as shown in Fig. 3. The coefficient of friction between the log and the ground is 0.6 . The $\log$ is modelled as a uniform rod and the cable as light.
(a) Show that the normal reaction on the $\log$ at $A$ is $\frac{2}{5} W$.
(b) Find the value of $\beta$.

The tension in the cable is $k W$.
(c) Find the value of $k$.

\section*{6678 <br> Edexcel GCE <br> Mechanics M2 <br> Advanced/Advanced Subsidiary <br> Wednesday 22 January 2003 - Afternoon <br> Time: 1 hour 30 minutes <br> Materials required for examination $\quad$| Items included with question papers |
| :--- |
| Answer Book (AB16) |
| Mathematical Formulae (Lilac) |
| Graph Paper (ASG2) |


| Candidates may use any calculator EXCEPT those with the facility for symbolic |
| :--- |
| algebra, differentiation and/or integration. Thus candidates may NOT use calculators |
| such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP | algebra, differentiation and/or integration. Thus candidates may NOT use calculator Hewlett Packard H} 48G.

## Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of $g$ is required, take $g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions. Pages 7 and 8 are blank.

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. Three particles of mass $3 m, 5 m$ and $\lambda m$ are placed at points with coordinates $(4,0),(0,-3)$ and $(4,2)$ respectively. The centre of mass of the system of three particles is at $(2, k)$
(a) Show that $\lambda=2$.
(b) Calculate the value of $k$.
2. A car of mass 1000 kg is moving along a straight horizontal road with a constant acceleration of $f \mathrm{~m} \mathrm{~s}^{-2}$. The resistance to motion is modelled as a constant force of magnitude 1200 N . When the car is travelling at $12 \mathrm{~m} \mathrm{~s}^{-1}$, the power generated by the engine of the car is 24 kW .
(a) Calculate the value of $f$.

When the car is travelling at $14 \mathrm{~m} \mathrm{~s}^{-1}$, the engine is switched off and the car comes to rest, without braking, in a distance of $d$ metres. Assuming the same model for resistance,
(b) use the work-energy principle to calculate the value of $d$.
(c) Give a reason why the model used for the resistance to motion may not be realistic.

Figure 1


A uniform ladder $A B$, of mass $m$ and length $2 a$, has one end $A$ on rough horizontal ground. The other end $B$ rests against a smooth vertical wall. The ladder is in a vertical plane perpendicular to the wall. The ladder makes an angle $\alpha$ with the horizontal, where $\tan \alpha=\frac{4}{3}$. A child of mass $2 m$ stands on the ladder at $C$ where $A C=\frac{1}{2} a$, as shown in Fig. 1. The ladder and the child are in equilibrium.

By modelling the ladder as a rod and the child as a particle, calculate the least possible value of the coefficient of friction between the ladder and the ground.
4.

## Figure 2



Figure 2 shows a uniform lamina $A B C D E$ such that $A B D E$ is a rectangle, $B C=C D, A B=8 a$ and $A E=6 a$. The point $X$ is the mid-point of $B D$ and $X C=4 a$. The centre of mass of the lamina is at $G$.
(a) Show that $G X=\frac{44}{15} a$.

The mass of the lamina is $M$. A particle of mass $\lambda M$ is attached to the lamina at $C$. The lamina is suspended from $B$ and hangs freely under gravity with $A B$ horizontal.
(b) Find the value of $\lambda$.
5. A particle $P$ moves on the $x$-axis. The acceleration of $P$ at time $t$ seconds is $(4 t-8) \mathrm{m} \mathrm{s}^{-2}$, measured in the direction of $x$ increasing. The velocity of $P$ at time $t$ seconds is $v \mathrm{~m} \mathrm{~s}^{-1}$. Given that $v=6$ when $t=0$, find
(a) $v$ in terms of $t$,
(b) the distance between the two points where $P$ is instantaneously at rest.
$\qquad$
6. A smooth sphere $P$ of mass $2 m$ is moving in a straight line with speed $u$ on a smooth horizontal table. Another smooth sphere $Q$ of mass $m$ is at rest on the table. The sphere $P$ collides directly with $Q$. The coefficient of restitution between $P$ and $Q$ is $\frac{1}{3}$. The spheres are modelled as particles.
(a) Show that, immediately after the collision, the speeds of $P$ and $Q$ are $\frac{5}{9} u$ and $\frac{8}{9} u$ respectively.

After the collision, $Q$ strikes a fixed vertical wall which is perpendicular to the direction of motion of $P$ and $Q$. The coefficient of restitution between $Q$ and the wall is $e$. When $P$ and $Q$ collide again, $P$ is brought to rest.
(b) Find the value of $e$.
(c) Explain why there must be a third collision between $P$ and $Q$.
7.

Figure 3


A ball $B$ of mass 0.4 kg is struck by a bat at a point $O$ which is 1.2 m above horizontal ground The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are respectively horizontal and vertical. Immediately before being struck, $B$ has velocity $(-20 \mathbf{i}+4 \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}$. Immediately after being struck it has velocity $(15 \mathbf{i}+16 \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}$.

After $B$ has been struck, it moves freely under gravity and strikes the ground at the point $A$, as After B has been struck, it moves freely under gravity
shown in Fig. 3. The ball is modelled as a particle.
(a) Calculate the magnitude of the impulse exerted by the bat on $B$
(b) By using the principle of conservation of energy, or otherwise, find the speed of $B$ when it reaches $A$.
(c) Calculate the angle which the velocity of $B$ makes with the ground when $B$ reaches $A$.
(d) State two additional physical factors which could be taken into account in a refinement of the model of the situation which would make it more realistic.

## Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), you centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678) our surname, other name and signature.
Whenever a numerical value of $g$ is required, take $g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions. Pages 6,7 and 8 are blank.

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. A particle $P$ moves on the $x$-axis. At time $t$ seconds the velocity of $P$ is $v \mathrm{~m} \mathrm{~s}^{-1}$ in the direction of $x$ increasing, where $v=6 t-2 t^{2}$. When $t=0, P$ is at the origin $O$. Find the distance of $P$ from $O$ when $P$ comes to instantaneous rest after leaving $O$.
2. A tennis ball of mass 0.2 kg is moving with velocity $(-10 \mathbf{i}) \mathrm{m} \mathrm{s}^{-1}$ when it is struck by a tennis racket. Immediately after being struck, the ball has velocity $(15 \mathbf{i}+15 \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}$. Find
(a) the magnitude of the impulse exerted by the racket on the ball,
(b) the angle, to the nearest degree, between the vector $\mathbf{i}$ and the impulse exerted by the racket,
(c) the kinetic energy gained by the ball as a result of being struck.
3. 

Figure 1


A uniform lamina $A B C D$ is made by taking a uniform sheet of metal in the form of a rectangle $A B E D$, with $A B=3 a$ and $A D=2 a$, and removing the triangle $B C E$, where $C$ lies on $D E$ and $C E=a$, as shown in Fig. 1 .
(a) Find the distance of the centre of mass of the lamina from $A D$.

The lamina has mass $M$. A particle of mass $m$ is attached to the lamina at $B$. When the loaded lamina is freely suspended from the mid-point of $A B$, it hangs in equilibrium with $A B$ horizontal.
(b) Find $m$ in terms of $M$.
4.

Figure 2


A uniform steel girder $A B$, of mass 40 kg and length 3 m , is freely hinged at $A$ to a vertical wall. The girder is supported in a horizontal position by a steel cable attached to the girder at $B$. The other end of the cable is attached to the point $C$ vertically above $A$ on the wall, with $\angle A B C=\alpha$, where $\tan \alpha=\frac{3}{4}$. A load of mass 60 kg is suspended by another cable from the girder at the point $D$, where $A D=2 \mathrm{~m}$, as shown in Fig. 2. The girder remains horizontal and in equilibrium. The girder is modelled as a rod, and the cables as light inextensible strings.
(a) Show that the tension in the cable $B C$ is 980 N .
(b) Find the magnitude of the reaction on the girder at $A$.
(c) Explain how you have used the modelling assumption that the cable at $D$ is light.

## Figure 3



8 m

A ball is thrown from a point 4 m above horizontal ground. The ball is projected at an angle $\alpha$ above the horizontal, where $\tan \alpha=\frac{3}{4}$. The ball hits the ground at a point which is a horizontal distance 8 m from its point of projection, as shown in Fig. 3. The initial speed of the ball is $u \mathrm{~m} \mathrm{~s}^{-1}$ and the time of flight is $T$ seconds.
(a) Prove that $u T=10$.
(b) Find the value of $u$

As the ball hits the ground, its direction of motion makes an angle $\phi$ with the horizontal.
(c) Find $\tan \phi$
6. A girl and her bicycle have a combined mass of 64 kg . She cycles up a straight stretch of road which is inclined at an angle $\alpha$ to the horizontal, where $\sin \alpha=\frac{1}{14}$. She cycles at a constant speed of $5 \mathrm{~m} \mathrm{~s}^{-1}$. When she is cycling at this speed, the resistance to motion from non-gravitational forces has magnitude 20 N .
(a) Find the rate at which the cyclist is working.

She now turns round and comes down the same road. Her initial speed is $5 \mathrm{~m} \mathrm{~s}^{-1}$, and the resistance to motion is modelled as remaining constant with magnitude 20 N . She free-wheels down the road for a distance of 80 m . Using this model,
(b) find the speed of the cyclist when she has travelled a distance of 80 m .

The cyclist again moves down the same road, but this time she pedals down the road. The resistance is now modelled as having magnitude proportional to the speed of the cyclist. Her initial speed is again $5 \mathrm{~m} \mathrm{~s}^{-1}$ when the resistance to motion has magnitude 20 N .
(c) Find the magnitude of the resistance to motion when the speed of the cyclist is $8 \mathrm{~m} \mathrm{~s}^{-1}$.

The cyclist works at a constant rate of 200 W .
(d) Find the magnitude of her acceleration when her speed is $8 \mathrm{~m} \mathrm{~s}^{-1}$.
7. A uniform sphere $A$ of mass $m$ is moving with speed $u$ on a smooth horizontal table when it collides directly with another uniform sphere $B$ of mass $2 m$ which is at rest on the table. The spheres are of equal radius and the coefficient of restitution between them is $e$. The direction of motion of $A$ is unchanged by the collision.
(a) Find the speeds of $A$ and $B$ immediately after the collision.
(b) Find the range of possible values of $e$.

After being struck by $A$, the sphere $B$ collides directly with another sphere $C$, of mass $4 m$ and of the same size as $B$. The sphere $C$ is at rest on the table immediately before being struck by $B$. The coefficient of restitution between $B$ and $C$ is also $e$.
(c) Show that, after $B$ has struck $C$, there will be a further collision between $A$ and $B$.

## 6678 <br> Edexcel GCE

## Mechanics M2 <br> Advanced/Advanced Subsidiary

Wednesday 21 January 2004 - Afternoon
Time: $\mathbf{1}$ hour 30 minutes

Materials required for examination
Answer Book (AB16)
Mathematical Formulae (Lilac)
Graph Paper (ASG2)
$\frac{\text { Item }}{\text { Nil }}$

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

## Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your center number, candid , it (M2), , it surname, other name and signature
Whenever a numerical value of $g$ is required, take $g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions
This paper has seven questions

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. A car of mass 400 kg is moving up a straight road inclined at an angle $\theta$ to the horizontal, where $\sin \theta=\frac{1}{14}$. The resistance to motion of the car from non-gravitational forces is modelled as a constant force of magnitude $R$ newtons. When the car is moving at a constant speed of $20 \mathrm{~m} \mathrm{~s}^{-1}$, the power developed by the car's engine is 10 kW .

Find the value of $R$.
2. A particle $P$ of mass 0.75 kg is moving under the action of a single force $\mathbf{F}$ newtons. At time $t$ seconds, the velocity $\mathbf{v} \mathrm{m} \mathrm{s}^{-1}$ of $P$ is given by

$$
\mathbf{v}=\left(t^{2}+2\right) \mathbf{i}-6 \mathbf{t} \mathbf{j} .
$$

(a) Find the magnitude of $\mathbf{F}$ when $t=4$

When $t=5$, the particle $P$ receives an impulse of magnitude $9 \sqrt{ } 2 \mathrm{Ns}$ in the direction of the vector $\mathbf{i}-\mathbf{j}$.
(b) Find the velocity of $P$ immediately after the impulse.

N13537A

## Figure 1



A particle $P$ of mass 2 kg is projected from a point $A$ up a line of greatest slope $A B$ of a fixed plane. The plane is inclined at an angle of $30^{\circ}$ to the horizontal and $A B=3 \mathrm{~m}$ with $B$ above $A$, as shown in Fig. 1. The speed of $P$ at $A$ is $10 \mathrm{~m} \mathrm{~s}^{-1}$.

Assuming the plane is smooth,
(a) find the speed of $P$ at $B$.

The plane is now assumed to be rough. At $A$ the speed of $P$ is $10 \mathrm{~m} \mathrm{~s}^{-1}$ and at $B$ the speed of $P$ is $7 \mathrm{~m} \mathrm{~s}^{-1}$. By using the work-energy principle, or otherwise,
(b) find the coefficient of friction between $P$ and the plane.

五

Figure 2


A uniform ladder, of weight $W$ and length $2 a$, rests in equilibrium with one end $A$ on a smooth horizontal floor and the other end $B$ on a rough vertical wall. The ladder is in a vertical plane perpendicular to the wall. The coefficient of friction between the wall and the ladder is $\mu$. The adder makes an angle $\theta$ with the floor, where $\tan \theta=2$. A horizontal light inextensible string $C D$ is attached to the ladder at the point $C$, where $A C=\frac{1}{2} a$. The string is attached to the wall at the point $D$, with $B D$ vertical, as shown in Fig. 2. The tension in the string is $\frac{1}{4} W$. By modelling the ladder as a rod,
(a) find the magnitude of the force of the floor on the ladder,
(b) show that $\mu \geq \frac{1}{2}$.
(c) State how you have used the modelling assumption that the ladder is a rod.
5. A particle $P$ is projected with velocity $(2 u \mathbf{i}+3 u \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}$ from a point $O$ on a horizontal plane, where $\mathbf{i}$ and $\mathbf{j}$ are horizontal and vertical unit vectors respectively. The particle $P$ strikes the plane at the point $A$ which is 735 m from $O$.
(a) Show that $u=24.5$.
(b) Find the time of flight from $O$ to $A$.

The particle $P$ passes through a point $B$ with speed $65 \mathrm{~m} \mathrm{~s}^{-1}$.
(c) Find the height of $B$ above the horizontal plane.
6. A smooth sphere $A$ of mass $m$ is moving with speed $u$ on a smooth horizontal table when it collides directly with another smooth sphere $B$ of mass 3 m , which is at rest on the table. The coefficient of restitution between $A$ and $B$ is $e$. The spheres have the same radius and are modelled as particles.
(a) Show that the speed of $B$ immediately after the collision is $\frac{1}{4}(1+e) u$.
(b) Find the speed of $A$ immediately after the collision.

Immediately after the collision the total kinetic energy of the spheres is $\frac{1}{6} m u^{2}$
(c) Find the value of $e$
(d) Hence show that $A$ is at rest after the collision.

Figure 3


A loaded plate $L$ is modelled as a uniform rectangular lamina $A B C D$ and three particles. The sides $C D$ and $A D$ of the lamina have lengths $5 a$ and $2 a$ respectively and the mass of the lamina is $3 m$. The three particles have mass $4 m, m$ and $2 m$ and are attached at the points $A, B$ and $C$ respectively, as shown in Fig. 3.
(a) Show that the distance of the centre of mass of $L$ from $A D$ is $2.25 a$.
(b) Find the distance of the centre of mass of $L$ from $A B$.

The point $O$ is the mid-point of $A B$. The loaded plate $L$ is freely suspended from $O$ and hangs at rest under gravity.
(c) Find, to the nearest degree, the size of the angle that $A B$ makes with the horizontal.

A horizontal force of magnitude $P$ is applied at $C$ in the direction $C D$. The loaded plate $L$ remains suspended from $O$ and rests in equilibrium with $A B$ horizontal and $C$ vertically below $B$.
(d) Show that $P=\frac{5}{4} \mathrm{mg}$
(e) Find the magnitude of the force on $L$ at $O$

## 6678 <br> Edexcel GCE

Mechanics M2
Advanced/Advanced Subsidiary
Friday 25 June 2004 - Morning
Time: 1 hour 30 minutes
Materials required for examination $\quad$ Items included with question papers
Answer Book (AB16)
Mathematical Formulae (Lilac)
Graph Paper (ASG2)

Candidates may use any calculator EXCEPT those with the facility for symbolic
algebra, differentiation and/or integration. Thus candidates may NOT use calculators
such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard
HP 48G.
algebra, differentiation and/or integration. Thus candidates may NOT use calculators
such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

## Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of $g$ is required, take $g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided
Full marks may be obtained for answers to ALL questions.
This paper has seven questions
$\underline{\text { Advice to Candidates }}$
You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answer without working may gain no credit.

1. A lorry of mass 1500 kg moves along a straight horizontal road. The resistance to the motion of the lorry has magnitude 750 N and the lorry's engine is working at a rate of 36 kW .
(a) Find the acceleration of the lorry when its speed is $20 \mathrm{~m} \mathrm{~s}^{-1}$.

The lorry comes to a hill inclined at an angle $\alpha$ to the horizontal, where $\sin \alpha=\frac{1}{10}$. The magnitude of the resistance to motion from non-gravitational forces remains 750 N .

The lorry moves up the hill at a constant speed of $20 \mathrm{~m} \mathrm{~s}^{-1}$.
(b) Find the rate at which the lorry's engine is now working.
2. [In this question $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors in a horizontal plane.]

A ball has mass 0.2 kg . It is moving with velocity ( $30 \mathbf{i}$ ) $\mathrm{m} \mathrm{s}^{-1}$ when it is struck by a bat. The bat exerts an impulse of $(-4 \mathbf{i}+4 \mathbf{j}) \mathrm{Ns}$ on the ball.

Find
(a) the velocity of the ball immediately after the impact,
(b) the angle through which the ball is deflected as a result of the impact,
(c) the kinetic energy lost by the ball in the impact.
3.

Figure 1


Figure 1 shows a decoration which is made by cutting the shape of a simple tree from a sheet of uniform card. The decoration consists of a triangle $A B C$ and a rectangle $P Q R S$. The points $P$ and $S$ lie on $B C$ and $M$ is the mid-point of both $B C$ and $P S$. The triangle $A B C$ is isosceles with $A B=A C, B C=4 \mathrm{~cm}, A M=6 \mathrm{~cm}, P S=2 \mathrm{~cm}$ and $P Q=3 \mathrm{~cm}$.
(a) Find the distance of the centre of mass of the decoration from $B C$.

The decoration is suspended from $Q$ and hangs freely.
(b) Find, in degrees to one decimal place, the angle between $P Q$ and the vertical
4. At time $t$ seconds, the velocity of a particle $P$ is $[(4 t-7) \mathbf{i}-5 \mathbf{j}] \mathrm{m} \mathrm{s}^{-1}$. When $t=0, P$ is at the point with position vector $(3 \mathbf{i}+5 \mathbf{j}) \mathrm{m}$ relative to a fixed origin $O$.
(a) Find an expression for the position vector of $P$ after $t$ seconds, giving your answer in the form $(a \mathbf{i}+b \mathbf{j}) \mathrm{m}$.

A second particle $Q$ moves with constant velocity $(2 \mathbf{i}-3 \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}$. When $t=0$, the position vector of $Q$ is $(-7 \mathbf{i}) \mathrm{m}$.
(b) Prove that $P$ and $Q$ collide.
5. Two small smooth spheres, $P$ and $Q$, of equal radius, have masses $2 m$ and $3 m$ respectively. The sphere $P$ is moving with speed $5 u$ on a smooth horizontal table when it collides directly with $Q$, which is at rest on the table. The coefficient of restitution between $P$ and $Q$ is $e$.
(a) Show that the speed of $Q$ immediately after the collision is $2(1+e) u$.

After the collision, $Q$ hits a smooth vertical wall which is at the edge of the table and perpendicular to the direction of motion of $Q$. The coefficient of restitution between $Q$ and the wall is $f, 0<f \leq 1$.
(b) Show that, when $e=0.4$, there is a second collision between $P$ and $Q$.

Given that $e=0.8$ and there is a second collision between $P$ and $Q$,
(c) find the range of possible values of $f$.
6. A uniform ladder $A B$, of mass $m$ and length $2 a$, has one end $A$ on rough horizontal ground. The coefficient of friction between the ladder and the ground is 0.6 . The other end $B$ of the ladder rests against a smooth vertical wall.
A builder of mass 10 m stands at the top of the ladder. To prevent the ladder from slipping, the builder's friend pushes the bottom of the ladder horizontally towards the wall with a force of magnitude $P$. This force acts in a direction perpendicular to the wall. The ladder rests in equilibrium in a vertical plane perpendicular to the wall and makes an angle $\alpha$ with the horizontal, where $\tan \alpha=\frac{3}{2}$.
(a) Show that the reaction of the wall on the ladder has magnitude 7 mg .
(b) Find, in terms of $m$ and $g$, the range of values of $P$ for which the ladder remains in equilibrium.
7.


In a ski-jump competition, a skier of mass 80 kg moves from rest at a point $A$ on a ski-slope. The skier's path is an arc $A B$. The starting point $A$ of the slope is 32.5 m above horizontal ground. The end $B$ of the slope is 8.1 m above the ground. When the skier reaches $B$, she is travelling at $20 \mathrm{~m} \mathrm{~s}^{-1}$, and moving upwards at an angle $\alpha$ to the horizontal, where $\tan \alpha=\frac{3}{4}$, as shown in Fig. 2 The distance along the slope from $A$ to $B$ is 60 m . The resistance to motion while she is on the slope is modelled as a force of constant magnitude $R$ newtons. By using the work-energy principle,
(a) find the value of $R$.

On reaching $B$, the skier then moves through the air and reaches the ground at the point $C$. The motion of the skier in moving from $B$ to $C$ is modelled as that of a particle moving freely under gravity.
(b) Find the time for the skier to move from $B$ to $C$.
(c) Find the horizontal distance from $B$ to $C$.
(d) Find the speed of the skier immediately before she reaches $C$.

## 6678 <br> Edexcel GCE

## Mechanics M2

## Advanced Subsidiary

Wednesday 12 January 2005 - Afternoon
Time: 1 hour 30 minutes

```
Materials required for examination
athematical Formulae (Lilac or Green)
\(\frac{\text { Items included with question papers }}{\text { Nil }}\)
```

the facility for symb algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

## Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature
Whenever a numerical value of $g$ is required, take $g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions.
The total mark for this paper is 75

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.
1.

## Figure 1



A uniform rod $A B$, of length $8 a$ and weight $W$, is free to rotate in a vertical plane about a smooth pivot at $A$. One end of a light inextensible string is attached to $B$. The other end is attached to point $C$ which is vertically above $A$, with $A C=6 a$. The rod is in equilibrium with $A B$ horizontal, as shown in Figure 1
(a) By taking moments about $A$, or otherwise, show that the tension in the string is $\frac{5}{6} W$.
(b) Calculate the magnitude of the horizontal component of the force exerted by the pivot on the rod.
2.

## Figure 2



Figure 2 shows a metal plate that is made by removing a circle of centre $O$ and radius 3 cm from a uniform rectangular lamina $A B C D$, where $A B=20 \mathrm{~cm}$ and $B C=10 \mathrm{~cm}$. The point $O$ is 5 cm from both $A B$ and $C D$ and is 6 cm from $A D$.
(a) Calculate, to 3 significant figures, the distance of the centre of mass of the plate from $A D$.

The plate is freely suspended from $A$ and hangs in equilibrium.
(b) Calculate, to the nearest degree, the angle between $A B$ and the vertical.
3.

## Figure 3



A small package $P$ is modelled as a particle of mass 0.6 kg . The package slides down a rough plane from a point $S$ to a point $T$, where $S T=12 \mathrm{~m}$. The plane is inclined at an angle of $30^{\circ}$ to the horizontal and $S T$ is a line of greatest slope of the plane, as shown in Figure 3. The speed of $P$ at $S$ is $10 \mathrm{~m} \mathrm{~s}^{-1}$ and the speed of $P$ at $T$ is $9 \mathrm{~m} \mathrm{~s}^{-1}$. Calculate
(a) the total loss of energy of $P$ in moving from $S$ to $T$,
(b) the coefficient of friction between $P$ and the plane
4. A particle $P$ of mass 0.4 kg is moving under the action of a single force $\mathbf{F}$ newtons. At time $t$ seconds, the velocity of $P, \mathrm{v} \mathrm{m} \mathrm{s}^{-1}$, is given by

$$
\mathbf{v}=(6 t+4) \mathbf{i}+\left(t^{2}+3 t\right) \mathbf{j} .
$$

When $t=0, P$ is at the point with position vector $(-3 \mathbf{i}+4 \mathbf{j}) \mathrm{m}$. When $t=4, P$ is at the point $S$.
(a) Calculate the magnitude of $\mathbf{F}$ when $t=4$.
(b) Calculate the distance $O S$.
5. A car of mass 1000 kg is towing a trailer of mass 1500 kg along a straight horizontal road. The tow-bar joining the car to the trailer is modelled as a light rod parallel to the road. The total resistance to motion of the car is modelled as having constant magnitude 750 N . The total resistance to motion of the trailer is modelled as of magnitude $R$ newtons, where $R$ is a constant. When the engine of the car is working at a rate of 50 kW , the car and the trailer travel at a constant speed of $25 \mathrm{~m} \mathrm{~s}^{-1}$
(a) Show that $R=1250$

When travelling at $25 \mathrm{~m} \mathrm{~s}^{-1}$ the driver of the car disengages the engine and applies the brakes. The brakes provide a constant braking force of magnitude 1500 N to the car. The resisting forces of magnitude 750 N and 1250 N are assumed to remain unchanged. Calculate
(b) the deceleration of the car while braking,
(c) the thrust in the tow-bar while braking,
(d) the work done, in kJ , by the braking force in bringing the car and the trailer to rest.
(e) Suggest how the modelling assumption that the resistances to motion are constant could be refined to be more realistic.
6. A particle $P$ of mass $3 m$ is moving with speed $2 u$ in a straight line on a smooth horizontal table. The particle $P$ collides with a particle $Q$ of mass $2 m$ moving with speed $u$ in the opposite direction to $P$. The coefficient of restitution between $P$ and $Q$ is $e$.
(a) Show that the speed of $Q$ after the collision is $\frac{1}{5} u(9 e+4)$.

As a result of the collision, the direction of motion of $P$ is reversed.
(b) Find the range of possible values of $e$.

Given that the magnitude of the impulse of $P$ on $Q$ is $\frac{32}{5} m u$,
(c) find the value of $e$.

7
Figure 4

A particle $P$ is projected from a point $A$ with speed $32 \mathrm{~m} \mathrm{~s}^{-1}$ at an angle of elevation $\alpha$, where $\sin \alpha=\frac{3}{5}$. The point $O$ is on horizontal ground, with $O$ vertically below $A$ and $O A=20 \mathrm{~m}$. The particle $P$ moves freely under gravity and passes through a point $B$, which is 16 m above ground, before reaching the ground at the point $C$, as shown in Figure 4.

Calculate
(a) the time of the flight from $A$ to $C$,
(b) the distance $O C$,
(c) the speed of $P$ at $B$,
(d) the angle that the velocity of $P$ at $B$ makes with the horizontal.

$\qquad$

## 6678/01 <br> Edexcel GCE

## Mechanics M2

## Advanced/Advanced Subsidiary

Friday 24 June 2005 - Morning
Time: $\mathbf{1}$ hour 30 minutes

## Materials required for examination athematical Formulae (Lilac or Green) <br> Items included with question papers

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G Hewlett Packard HP 48G.

## Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), you centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature
Whenever a numerical value of $g$ is required, take $g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information for Candidate

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has 7 questions.
The total mark for this paper is 75

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. A car of mass 1200 kg moves along a straight horizontal road. The resistance to motion of the car from non-gravitational forces is of constant magnitude 600 N . The car moves with constant speed and the engine of the car is working at a rate of 21 kW .
(a) Find the speed of the car.

The car moves up a hill inclined at an angle $\alpha$ to the horizontal, where $\sin \alpha=\frac{1}{14}$.
The car's engine continues to work at 21 kW and the resistance to motion from nongravitational forces remains of magnitude 600 N .
(b) Find the constant speed at which the car can move up the hill.

## Figure 1



A thin uniform wire, of total length 20 cm , is bent to form a frame. The frame is in the shape of a trapezium $A B C D$, where $A B=A D=4 \mathrm{~cm}, C D=5 \mathrm{~cm}$ and $A B$ is perpendicular to $B C$ and $A D$, as shown in Figure 1.
(a) Find the distance of the centre of mass of the frame from $A B$

The frame has mass $M$. A particle of mass $k M$ is attached to the frame at $C$. When the frame is freely suspended from the mid-point of $B C$, the frame hangs in equilibrium with $B C$ horizontal.
(b) Find the value of $k$.
3. A particle $P$ moves in a horizontal plane. At time $t$ seconds, the position vector of $P$ is $\mathbf{r}$ metres relative to a fixed origin $O$, and $\mathbf{r}$ is given by

$$
\mathbf{r}=\left(18 t-4 t^{3}\right) \mathbf{i}+c t^{2} \mathbf{j}
$$

where $c$ is a positive constant. When $t=1.5$, the speed of $P$ is $15 \mathrm{~m} \mathrm{~s}^{-1}$. Find
(a) the value of $c$,
(b) the acceleration of $P$ when $t=1.5$.
4. A darts player throws darts at a dart board which hangs vertically. The motion of a dart is modelled as that of a particle moving freely under gravity. The darts move in a vertical plane which is perpendicular to the plane of the dart board. A dart is thrown horizontally with speed $12.6 \mathrm{~m} \mathrm{~s}^{-1}$. It hits the board at a point which is 10 cm below the level from which it was thrown.
a) Find the horizontal distance from the point where the dart was thrown to the dart board.

The darts player moves his position. He now throws a dart from a point which is at a horizontal distance of 2.5 m from the board. He throws the dart at an angle of elevation $\alpha$ to the horizontal, where $\tan \alpha=\frac{7}{24}$. This dart hits the board at a point which is at the same level as the point from which it was thrown.
(b) Find the speed with which the dart is thrown.
5. Two small spheres $A$ and $B$ have mass $3 m$ and $2 m$ respectively. They are moving towards each other in opposite directions on a smooth horizontal plane, both with speed $2 u$, when they collide directly. As a result of the collision, the direction of motion of $B$ is reversed and its speed is unchanged.
(a) Find the coefficient of restitution between the spheres

Subsequently, $B$ collides directly with another small sphere $C$ of mass $5 m$ which is at rest. The coefficient of restitution between $B$ and $C$ is $\frac{3}{5}$.
(b) Show that, after $B$ collides with $C$, there will be no further collisions between the spheres.
6.

## Figure 2



A uniform pole $A B$, of mass 30 kg and length 3 m , is smoothly hinged to a vertical wall at one end $A$. The pole is held in equilibrium in a horizontal position by a light rod $C D$. One end $C$ of the rod is fixed to the wall vertically below $A$. The other end $D$ is freely jointed to the pole so that $\angle A C D=30^{\circ}$ and $A D=0.5 \mathrm{~m}$, as shown in Figure 2. Find
(a) the thrust in the rod $C D$,
(b) the magnitude of the force exerted by the wall on the pole at $A$.

The rod $C D$ is removed and replaced by a longer light rod $C M$, where $M$ is the mid-point of $A B$. The rod is freely jointed to the pole at $M$. The pole $A B$ remains in equilibrium in a horizontal position.
(c) Show that the force exerted by the wall on the pole at $A$ now acts horizontally.
7. At a demolition site, bricks slide down a straight chute into a container. The chute is rough and is inclined at an angle of $30^{\circ}$ to the horizontal. The distance travelled down the chute by each brick is 8 m . A brick of mass 3 kg is released from rest at the top of the chute. When it reaches the bottom of the chute, its speed is $5 \mathrm{~m} \mathrm{~s}^{-1}$.
(a) Find the potential energy lost by the brick in moving down the chute.
(b) By using the work-energy principle, or otherwise, find the constant frictional force acting on the brick as it moves down the chute.
(c) Hence find the coefficient of friction between the brick and the chute.

Another brick of mass 3 kg slides down the chute. This brick is given an initial speed of $2 \mathrm{~m} \mathrm{~s}^{-1}$ at the top of the chute
(d) Find the speed of this brick when it reaches the bottom of the chute.

## 6678/01 <br> Edexcel GCE

## Mechanics M2

# Advanced /Advanced Subsidiary <br> Thursday 12 January 2006 - Afternoon <br> Time: $\mathbf{1}$ hour 30 minutes 

$\frac{\text { Materials required for examination }}{\text { Mathematical Formulae (Green or Lilac) }}$
$\frac{\text { Items included with question papers }}{\mathrm{Nil}}$

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G

## Instructions to Candidate

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature
Whenever a numerical value of $g$ is required, take $g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables’ is provided
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 7 questions on this paper. The total mark for this paper is 75 .

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. A brick of mass 3 kg slides in a straight line on a horizontal floor. The brick is modelled as a particle and the floor as a rough plane. The initial speed of the brick is $8 \mathrm{~m} \mathrm{~s}^{-1}$. The brick is brought to rest after moving 12 m by the constant frictional force between the brick and the floor.
(a) Calculate the kinetic energy lost by the brick in coming to rest, stating the units of your answer.
(b) Calculate the coefficient of friction between the brick and the floor.
2. A particle $P$ of mass 0.4 kg is moving so that its position vector $\mathbf{r}$ metres at time $t$ seconds is given by

$$
\mathbf{r}=\left(t^{2}+4 t\right) \mathbf{i}+\left(3 t-t^{3}\right) \mathbf{j} .
$$

(a) Calculate the speed of $P$ when $t=3$.

When $t=3$, the particle $P$ is given an impulse $(8 \mathbf{i}-12 \mathbf{j}) \mathrm{N}$ s.
(b) Find the velocity of $P$ immediately after the impulse
3. A car of mass 1000 kg is moving along a straight horizontal road. The resistance to motion is modelled as a constant force of magnitude $R$ newtons. The engine of the car is working at a rate of 12 kW . When the car is moving with speed $15 \mathrm{~m} \mathrm{~s}^{-1}$, the acceleration of the car s $0.2 \mathrm{~m} \mathrm{~s}^{-2}$.
(a) Show that $R=600$.

The car now moves with constant speed $U \mathrm{~m} \mathrm{~s}^{-1}$ downhill on a straight road inclined at $\theta$ to the horizontal, where $\sin \theta=\frac{1}{40}$. The engine of the car is now working at a rate of 7 kW . The resistance to motion from non-gravitational forces remains of magnitude $R$ newtons.
(c) Calculate the value of $U$.
4. A particle $A$ of mass $2 m$ is moving with speed $3 u$ in a straight line on a smooth horizontal table. The particle collides directly with a particle $B$ of mass $m$ moving with speed $2 u$ in the opposite direction to $A$. Immediately after the collision the speed of $B$ is $\frac{8}{3} u$ and the direction of motion of $B$ is reversed.
(a) Calculate the coefficient of restitution between $A$ and $B$
(b) Show that the kinetic energy lost in the collision is $7 m u^{2}$.

After the collision $B$ strikes a fixed vertical wall that is perpendicular to the direction of motion of $B$. The magnitude of the impulse of the wall on $B$ is $\frac{14}{3} \mathrm{mu}$.
(c) Calculate the coefficient of restitution between $B$ and the wall.
5.


Figure 1 shows a triangular lamina $A B C$. The coordinates of $A, B$ and $C$ are $(0,4),(9,0)$ and $(0,-4)$ respectively. Particles of mass $4 m, 6 m$ and $2 m$ are attached at $A, B$ and $C$ respectively.
(a) Calculate the coordinates of the centre of mass of the three particles, without the lamina.

The lamina $A B C$ is uniform and of mass $k m$. The centre of mass of the combined system consisting of the three particles and the lamina has coordinates $(4, \lambda)$.
(b) Show that $k=6$.
(c) Calculate the value of $\lambda$.

The combined system is freely suspended from $O$ and hangs at rest.
(c) Calculate, in degrees to one decimal place, the angle between $A C$ and the vertical.

Figure 2


A ladder $A B$, of weight $W$ and length $4 a$, has one end $A$ on rough horizontal ground. The coefficient of friction between the ladder and the ground is $\mu$. The other end $B$ rests against a smooth vertical wall. The ladder makes an angle $\theta$ with the horizontal, where $\tan \theta=2$. A load of weight $4 W$ is placed at the point $C$ on the ladder, where $A C=3 a$, as shown in Figure 2. The ladder is modelled as a uniform rod which is in a vertical plane perpendicular to the wall. The load is modelled as a particle. Given that the system is in limiting equilibrium,
(a) show that $\mu=0.35$.

A second load of weight $k W$ is now placed on the ladder at $A$. The load of weight $4 W$ is removed from $C$ and placed on the ladder at $B$. The ladder is modelled as a uniform rod which is in a vertical plane perpendicular to the wall. The loads are modelled as particles. Given that the ladder and the loads are in equilibrium,
(b) find the range of possible values of $k$.

## Figure 3



The object of a game is to throw a ball $B$ from a point $A$ to hit a target $T$ which is placed at the top of a vertical pole, as shown in Figure 3. The point $A$ is 1 m above horizontal ground and the height of the pole is 2 m . The pole is at a horizontal distance of 10 m from $A$. The ball $B$ is projected from $A$ with a speed of $11 \mathrm{~m} \mathrm{~s}^{-1}$ at an angle of elevation of $30^{\circ}$. The ball hits the pole at the point $C$. The ball $B$ and the target $T$ are modelled as particles.
(a) Calculate, to 2 decimal places, the time taken for $B$ to move from $A$ to $C$.
(b) Show that $C$ is approximately 0.63 m below $T$.

The ball is thrown again from $A$. The speed of projection of $B$ is increased to $V \mathrm{~m} \mathrm{~s}^{-1}$, the angle of elevation remaining $30^{\circ}$. This time $B$ hits $T$.
(c) Calculate the value of $V$.
(d) Explain why, in practice, a range of values of $V$ would result in $B$ hitting the target.

TOTAL FOR PAPER: 75 MARKS
END

## 6678/01 <br> Edexcel GCE

## Mechanics M2

## Advanced Subsidiary

Tuesday 6 June 2006 - Afternoon
Time: $\mathbf{1}$ hour 30 minutes
$\frac{\text { Materials required for examination }}{\text { Mathematical Formulae (Lilac or Green) }}$
$\frac{\text { Items included with question papers }}{\text { Nil }}$

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

## Instructions to Candidate

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature
Whenever a numerical value of $g$ is required, take $g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided
Full marks may be obtained for answers to ALL questions.
This paper has 8 questions.
The total mark for this paper is 75 .

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. A particle $P$ moves on the $x$-axis. At time $t$ seconds, its acceleration is $(5-2 t) \mathrm{m} \mathrm{s}^{-2}$, measured in the direction of $x$ increasing. When $t=0$, its velocity is $6 \mathrm{~m} \mathrm{~s}^{-1}$ measured in the direction of $x$ increasing. Find the time when $P$ is instantaneously at rest in the subsequent motion.
2. A car of mass 1200 kg moves along a straight horizontal road with a constant speed of $24 \mathrm{~m} \mathrm{~s}^{-1}$. The resistance to motion of the car has magnitude 600 N
(a) Find, in kW , the rate at which the engine of the car is working

The car now moves up a hill inclined at $\alpha$ to the horizontal, where $\sin \alpha=\frac{1}{28}$. The resistance to motion of the car from non-gravitational forces remains of magnitude 600 N . The engine of the car now works at a rate of 30 kW .
(b) Find the acceleration of the car when its speed is $20 \mathrm{~m} \mathrm{~s}^{-1}$.
3. A cricket ball of mass 0.5 kg is struck by a bat. Immediately before being struck, the velocity of the ball is $(-30 \mathbf{i}) \mathrm{m} \mathrm{s}^{-1}$. Immediately after being struck, the velocity of the ball is $(16 \mathbf{i}+20 \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}$
(a) Find the magnitude of the impulse exerted on the ball by the bat

In the subsequent motion, the position vector of the ball is $\mathbf{r}$ metres at time $t$ seconds. In a model of the situation, it is assumed that $\mathbf{r}=\left[16 t \mathbf{i}+\left(20 t-5 t^{2}\right) \mathbf{j}\right]$. Using this model,
b) find the speed of the ball when $t=3$
4.

Figure 1


Figure 1 shows four uniform rods joined to form a rigid rectangular framework $A B C D$, where $A B=C D=2 a$, and $B C=A D=3 a$. Each rod has mass $m$. Particles, of mass $6 m$ and $2 m$, are attached to the framework at points $C$ and $D$ respectively.
(a) Find the distance of the centre of mass of the loaded framework from
(i) $A B$,
(ii) $A D$.

The loaded framework is freely suspended from $B$ and hangs in equilibrium.
(b) Find the angle which $B C$ makes with the vertical.
5. A vertical cliff is 73.5 m high. Two stones $A$ and $B$ are projected simultaneously. Stone $A$ is projected horizontally from the top of the cliff with speed $28 \mathrm{~m} \mathrm{~s}^{-1}$. Stone $B$ is projected from the bottom of the cliff with speed $35 \mathrm{~m} \mathrm{~s}^{-1}$ at an angle $\alpha$ above the horizontal. The stones move freely under gravity in the same vertical plane and collide in mid-air. By considering the horizontal motion of each stone,
(a) prove that $\cos \alpha=\frac{4}{5}$.
(b) Find the time which elapses between the instant when the stones are projected and the instant when they collide.
6.

Figure 2


A wooden plank $A B$ has mass $4 m$ and length $4 a$. The end $A$ of the plank lies on rough horizontal ground. A small stone of mass $m$ is attached to the plank at $B$. The plank is resting horizontal ground. A small stone of mass $m$ is attached to the plank at $B$. The plank is resting
on a small smooth horizontal peg $C$, where $B C=a$, as shown in Figure 2. The plank is in equilibrium making an angle $\alpha$ with the horizontal, where $\tan \alpha=\frac{3}{4}$. The coefficient of friction between the plank and the ground is $\mu$. The plank is modelled as a uniform rod lying in a vertical plane perpendicular to the peg, and the stone as a particle. Show that
(a) the reaction of the peg on the plank has magnitude $\frac{16}{5} \mathrm{mg}$,
(b) $\mu \geq \frac{48}{61}$.
(c) State how you have used the information that the peg is smooth
7. A particle $P$ has mass 4 kg . It is projected from a point $A$ up a line of greatest slope of a rough plane inclined at an angle $\alpha$ to the horizontal, where $\tan \alpha=\frac{3}{4}$. The coefficient of friction between $P$ and the plane is $\frac{2}{7}$. The particle comes to rest instantaneously at the point $B$ on the plane, where $A B=2.5 \mathrm{~m}$. It then moves back down the plane to $A$.
(a) Find the work done by friction as $P$ moves from $A$ to $B$.
(b) Using the work-energy principle, find the speed with which $P$ is projected from $A$.
(c) Find the speed of $P$ when it returns to $A$.
8. Two particles $A$ and $B$ move on a smooth horizontal table. The mass of $A$ is $m$, and the mass of $B$ is $4 m$. Initially $A$ is moving with speed $u$ when it collides directly with $B$, which is at rest on the table. As a result of the collision, the direction of motion of $A$ is reversed. The coefficient of restitution between the particles is $e$.
(a) Find expressions for the speed of $A$ and the speed of $B$ immediately after the collision.
(7)

In the subsequent motion, $B$ strikes a smooth vertical wall and rebounds. The wall is perpendicular to the direction of motion of $B$. The coefficient of restitution between $B$ and the wall is $\frac{4}{5}$. Given that there is a second collision between $A$ and $B$,
(b) show that $\frac{1}{4}<e<\frac{9}{16}$.

Given that $e=\frac{1}{2}$,
(c) find the total kinetic energy lost in the first collision between $A$ and $B$.

## 6678/01 <br> Edexcel GCE

## Mechanics M2

Advanced /Advanced Subsidiary<br>Thursday 25 January 2007 - Morning<br>Time: $\mathbf{1}$ hour $\mathbf{3 0}$ minutes

## Materials required for examination Items included with question papers athematical Formulae (Green or Lilac)

Candiates may use any calculator EXCEPT Hose wis the facity for symboric algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

## Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature
Whenever a numerical value of $g$ is required, take $g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2)
There are 7 questions on this paper. The total mark for this paper is 75 .

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. A particle of mass 0.8 kg is moving in a straight line on a rough horizontal plane. The speed of the particle is reduced from $15 \mathrm{~m} \mathrm{~s}^{-1}$ to $10 \mathrm{~m} \mathrm{~s}^{-1}$ as the particle moves 20 m . Assuming the only resistance to motion is the friction between the particle and the plane, find
(a) the work done by friction in reducing the speed of the particle from $15 \mathrm{~m} \mathrm{~s}^{-1}$ to $10 \mathrm{~m} \mathrm{~s}^{-1}$,
(b) the coefficient of friction between the particle and the plane.
2. A car of mass 800 kg is moving at a constant speed of $15 \mathrm{~m} \mathrm{~s}^{-1}$ down a straight road inclined at an angle $\alpha$ to the horizontal, where $\sin \alpha=\frac{1}{24}$. The resistance to motion from non-gravitational forces is modelled as a constant force of magnitude 900 N
(a) Find, in kW , the rate of working of the engine of the car.

When the car is travelling down the road at $15 \mathrm{~m} \mathrm{~s}^{-1}$, the engine is switched off. The car comes to rest in time $T$ seconds after the engine is switched off. The resistance to motion from non-gravitational forces is again modelled as a constant force of magnitude 900 N .
(b) Find the value of $T$.
3.


Figure 1 shows a template $T$ made by removing a circular disc, of centre $X$ and radius 8 cm , from a uniform circular lamina, of centre $O$ and radius 24 cm . The point $X$ lies on the diameter $A O B$ of the lamina and $A X=16 \mathrm{~cm}$. The centre of mass of $T$ is at the point $G$.
(a) Find $A G$.

The template $T$ is free to rotate about a smooth fixed horizontal axis, perpendicular to the plane of $T$, which passes through the mid-point of $O B$. A small stud of mass $\frac{1}{4} m$ is fixed at $B$, and $T$ and the stud are in equilibrium with $A B$ horizontal. Modelling the stud as a particle,
(b) find the mass of $T$ in terms of $m$
4. A particle $P$ of mass $m$ is moving in a straight line on a smooth horizontal table. Another particle $Q$ of mass km is at rest on the table. The particle $P$ collides directly with $Q$. The direction of motion of $P$ is reversed by the collision. After the collision, the speed of $P$ is $v$ and the speed of $Q$ is 3 v . The coefficient of restitution between $P$ and $Q$ is $\frac{1}{2}$.
(a) Find, in terms of $v$ only, the speed of $P$ before the collision.
(b) Find the value of $k$.

After being struck by $P$, the particle $Q$ collides directly with a particle $R$ of mass 11 m which is at rest on the table. After this second collision, $Q$ and $R$ have the same speed and are moving in opposite directions. Show that
(c) the coefficient of restitution between $Q$ and $R$ is $\frac{3}{4}$,
(d) there will be a further collision between $P$ and $Q$.

## Figure 2



A horizontal uniform rod $A B$ has mass $m$ and length $4 a$. The end $A$ rests against a rough vertical wall. A particle of mass $2 m$ is attached to the rod at the point $C$, where $A C=3 a$. One end of a light inextensible string $B D$ is attached to the rod at $B$ and the other end is attached to the wall at a point $D$, where $D$ is vertically above $A$. The rod is in equilibrium in a vertical plane perpendicular to the wall. The string is inclined at an angle $\theta$ to the horizontal, where $\tan \theta=\frac{3}{4}$, as shown in Figure 2.
(a) Find the tension in the string.
(b) Show that the horizontal component of the force exerted by the wall on the rod has magnitude $\frac{8}{3} \mathrm{mg}$.

The coefficient of friction between the wall and the rod is $\mu$. Given that the rod is in limiting equilibrium,
(c) find the value of $\mu$.
6. A particle $P$ of mass 0.5 kg is moving under the action of a single force $\mathbf{F}$ newtons. At time $t$ seconds, $\mathbf{F}=\left(1.5 t^{2}-3\right) \mathbf{i}+2 t \mathbf{j}$. When $t=2$, the velocity of $P$ is $(-4 \mathbf{i}+5 \mathbf{j}) \mathrm{ms}^{-1}$.
(a) Find the acceleration of $P$ at time $t$ seconds.
(b) Show that, when $t=3$, the velocity of $P$ is $(9 \mathbf{i}+15 \mathbf{j}) \mathrm{ms}^{-1}$.

When $t=3$, the particle $P$ receives an impulse $\mathbf{Q N s}$. Immediately after the impulse the velocity of $P$ is $(-3 \mathbf{i}+20 \mathbf{j}) \mathrm{ms}^{-1}$. Find
(c) the magnitude of $\mathbf{Q}$,
(d) the angle between $\mathbf{Q}$ and $\mathbf{i}$.
7.


A particle $P$ is projected from a point $A$ with speed $u \mathrm{~m} \mathrm{~s}^{-1}$ at an angle of elevation $\theta$, where $\cos \theta=\frac{4}{5}$. The point $B$, on horizontal ground, is vertically below $A$ and $A B=45 \mathrm{~m}$. After projection, $P$ moves freely under gravity passing through point $C, 30 \mathrm{~m}$ above the ground, before striking the ground at the point $D$, as shown in Figure 3.

Given that $P$ passes through $C$ with speed $24.5 \mathrm{~m} \mathrm{~s}^{-1}$,
(a) using conservation of energy, or otherwise, show that $u=17.5$,
(b) find the size of the angle which the velocity of $P$ makes with the horizontal as $P$ passes through $C$,
(c) find the distance $B D$

TOTAL FOR PAPER: 75 MARKS

## 6678 <br> Edexcel GCE

## Mechanics M2

## Advanced Subsidiary

Thursday 7 June 2007 - Morning
Time: 1 hour 30 minutes

## Materials required for examination athematical Formulae (Green)

## Items included with question papers

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic Igebra manipulation, differentiation and integration, or have retrievable nathematical formulas stored in them.

## Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature
Whenever a numerical value of $g$ is required, take $g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information for Candidate

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper.
The total mark for this paper is 75 .

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. A cyclist and his bicycle have a combined mass of 90 kg . He rides on a straight road up a hill inclined at an angle $\alpha$ to the horizontal, where $\sin \alpha=\frac{1}{21}$. He works at a constant rate of 444 W and cycles up the hill at a constant speed of $6 \mathrm{~m} \mathrm{~s}^{-1}$.

Find the magnitude of the resistance to motion from non-gravitational forces as he cycles up the hill.
2. A particle $P$ of mass 0.5 kg moves under the action of a single force $\mathbf{F}$ newtons. At time $t$ seconds, the velocity $\mathbf{v} \mathrm{m} \mathrm{s}^{-1}$ of $P$ is given by

$$
\mathbf{v}=3 t^{2} \mathbf{i}+(1-4 t) \mathbf{j} .
$$

Find
(a) the acceleration of $P$ at time $t$ seconds,
(b) the magnitude of $\mathbf{F}$ when $t=2$.
3.

Figure 1


A uniform lamina $A B C D E F$ is formed by taking a uniform sheet of card in the form of a square $A X E F$, of side $2 a$, and removing the square $B X D C$ of side $a$, where $B$ and $D$ are the mid-points of $A X$ and $X E$ respectively, as shown in Figure 1.
(a) Find the distance of the centre of mass of the lamina from $A F$.

The lamina is freely suspended from $A$ and hangs in equilibrium.
(b) Find, in degrees to one decimal place, the angle which $A F$ makes with the vertical
4.

## Figure 2



Two particles $A$ and $B$, of mass $m$ and $2 m$ respectively, are attached to the ends of a light inextensible string. The particle $A$ lies on a rough plane inclined at an angle $\alpha$ to the horizontal, where $\tan \alpha=\frac{3}{4}$. The string passes over a small light smooth pulley $P$ fixed at the top of the plane. The particle $B$ hangs freely below $P$, as shown in Figure 2. The particles are released from rest with the string taut and the section of the string from $A$ to $P$ parallel to a line of greatest slope of the plane. The coefficient of friction between $A$ and the plane is $\frac{5}{8}$. When each particle has moved a distance $h, B$ has not reached the ground and $A$ has not reached $P$.
(a) Find an expression for the potential energy lost by the system when each particle has moved a distance $h$.

When each particle has moved a distance $h$, they are moving with speed $v$. Using the work-energy principle,
(b) find an expression for $v^{2}$, giving your answer in the form kgh , where k is a number.
5.

## Figure 3



A uniform beam $A B$ of mass 2 kg is freely hinged at one end $A$ to a vertical wall. The beam is held in equilibrium in a horizontal position by a rope which is attached to a point $C$ on the beam, where $A C=0.14 \mathrm{~m}$. The rope is attached to the point $D$ on the wall vertically above $A$, where $\angle A C D=30^{\circ}$, as shown in Figure 3. The beam is modelled as a uniform rod and the rope as a light inextensible string. The tension in the rope is 63 N .

Find
(a) the length of $A B$,
(b) the magnitude of the resultant reaction of the hinge on the beam at $A$
6.

Figure 4


A golf ball $P$ is projected with speed $35 \mathrm{~m} \mathrm{~s}^{-1}$ from a point $A$ on a cliff above horizontal ground. The angle of projection is $\alpha$ to the horizontal, where $\tan \alpha=\frac{4}{3}$. The ball moves freely under gravity and hits the ground at the point $B$, as shown in Figure 4.
(a) Find the greatest height of $P$ above the level of $A$.

The horizontal distance from $A$ to $B$ is 168 m .
(b) Find the height of $A$ above the ground.

By considering energy, or otherwise,
(c) find the speed of $P$ as it hits the ground at $B$.
7. Two small spheres $P$ and $Q$ of equal radius have masses $m$ and $5 m$ respectively. They lie on a smooth horizontal table. Sphere $P$ is moving with speed $u$ when it collides directly with sphere $Q$ which is at rest. The coefficient of restitution between the spheres is $e$, where $e>\frac{1}{5}$.
(a) (i) Show that the speed of $P$ immediately after the collision is $\frac{u}{6}(5 e-1)$.
(ii) Find an expression for the speed of $Q$ immediately after the collision, giving your answer in the form $\lambda u$, where $\lambda$ is in terms of $e$.

Three small spheres $A, B$ and $C$ of equal radius lie at rest in a straight line on a smooth horizontal table, with $B$ between $A$ and $C$. The spheres $A$ and $C$ each have mass $5 m$, and the mass of $B$ is $m$. Sphere $B$ is projected towards $C$ with speed $u$. The coefficient of restitution between each pair of spheres is $\frac{4}{5}$.
(b) Show that, after $B$ and $C$ have collided, there is a collision between $B$ and $A$.
(c) Determine whether, after $B$ and $A$ have collided, there is a further collision between $B$ and $C$.
8. A particle $P$ moves on the $x$-axis. At time $t$ seconds the velocity of $P$ is $v \mathrm{~m} \mathrm{~s}^{-1}$ in the direction of $x$ increasing, where $v$ is given by

$$
v= \begin{cases}8 t-\frac{3}{2} t^{2}, & 0 \leq t \leq 4 \\ 16-2 t, & t>4\end{cases}
$$

When $t=0, P$ is at the origin $O$.
Find
(a) the greatest speed of $P$ in the interval $0 \leq t \leq 4$,
(b) the distance of $P$ from $O$ when $t=4$,
(c) the time at which $P$ is instantaneously at rest for $t>4$,
(d) the total distance travelled by $P$ in the first 10 s of its motion.

## 6678/01 <br> Edexcel GCE

## Mechanics M2

## Advanced

Thursday 24 January 2008 - Morning
Time: 1 hour 30 minutes

## Materials required for examination athematical Formulae (Green) <br> tems included with question papers

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

## Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature
Whenever a numerical value of $g$ is required, take $g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information for Candidate

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2)
There are 7 questions on this paper. The total mark for this paper is 75 .

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. A parcel of mass 2.5 kg is moving in a straight line on a smooth horizontal floor. Initially the parcel is moving with speed $8 \mathrm{~m} \mathrm{~s}^{-1}$. The parcel is brought to rest in a distance of 20 m by a constant horizontal force of magnitude $R$ newtons. Modelling the parcel as a particle, find
(a) the kinetic energy lost by the parcel in coming to rest,
(b) the value of $R$.
2. At time $t$ seconds $(t \geq 0)$, a particle $P$ has position vector $\mathbf{p}$ metres, with respect to a fixed origin $O$, where

$$
\mathbf{p}=\left(3 t^{2}-6 t+4\right) \mathbf{i}+\left(3 t^{3}-4 t\right) \mathbf{j} .
$$

Find
(a) the velocity of $P$ at time $t$ seconds
(b) the value of $t$ when $P$ is moving parallel to the vector $\mathbf{i}$.

When $t=1$, the particle $P$ receives an impulse of $(2 \mathbf{i}-6 \mathbf{j}) \mathrm{N}$ s. Given that the mass of $P$ is 0.5 kg ,
(c) find the velocity of $P$ immediately after the impulse.
3. A car of mass 1000 kg is moving at a constant speed of $16 \mathrm{~m} \mathrm{~s}^{-1}$ up a straight road inclined a an angle $\theta$ to the horizontal. The rate of working of the engine of the car is 20 kW and the resistance to motion from non-gravitational forces is modelled as a constant force of magnitude 550 N
(a) Show that $\sin \theta=\frac{1}{14}$

When the car is travelling up the road at $16 \mathrm{~m} \mathrm{~s}^{-1}$, the engine is switched off. The car comes to rest, without braking, having moved a distance $y$ metres from the point where the engine was switched off. The resistance to motion from non-gravitational forces is again modelled as a constant force of magnitude 550 N
(b) Find the value of $y$.
4.


## Figure 1

A set square $S$ is made by removing a circle of centre $O$ and radius 3 cm from a triangular piece of wood. The piece of wood is modelled as a uniform triangular lamina $A B C$, with $\angle A B C=90^{\circ}, A B=12 \mathrm{~cm}$ and $B C=21 \mathrm{~cm}$. The point $O$ is 5 cm from $A B$ and 5 cm from $B C$, as shown in Figure 1
(a) Find the distance of the centre of mass of $S$ from
(i) $A B$,
(ii) $B C$.

The set square is freely suspended from $C$ and hangs in equilibrium.
(b) Find, to the nearest degree, the angle between $C B$ and the vertical.
5.


## Figure 2

A ladder $A B$, of mass $m$ and length $4 a$, has one end $A$ resting on rough horizontal ground. The other end $B$ rests against a smooth vertical wall. A load of mass 3 m is fixed on the ladder at the point $C$, where $A C=a$. The ladder is modelled as a uniform rod in a vertical plane perpendicular to the wall and the load is modelled as a particle. The ladder rests in limiting equilibrium making an angle of $30^{\circ}$ with the wall, as shown in Figure 2.

Find the coefficient of friction between the ladder and the ground.
6.


## Figure 3

[In this question, the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are in a vertical plane, $\mathbf{i}$ being horizontal and $\mathbf{j}$ being vertical.]

A particle $P$ is projected from the point $A$ which has position vector $47.5 \mathbf{j}$ metres with respect to a fixed origin $O$. The velocity of projection of $P$ is $(2 u \mathbf{i}+5 u \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}$. The particle moves freely under gravity passing through the point $B$ with position vector $30 \mathbf{i}$ metres, as shown in Figure 3.
(a) Show that the time taken for $P$ to move from $A$ to $B$ is 5 s .
(b) Find the value of $u$.
(c) Find the speed of $P$ at $B$.
7. A particle $P$ of mass $2 m$ is moving with speed $2 u$ in a straight line on a smooth horizontal plane. A particle $Q$ of mass 3 m is moving with speed $u$ in the same direction as $P$. The particles collide directly. The coefficient of restitution between $P$ and $Q$ is $\frac{1}{2}$
(a) Show that the speed of $Q$ immediately after the collision is $\frac{8}{5} u$.
(b) Find the total kinetic energy lost in the collision.

After the collision between $P$ and $Q$, the particle $Q$ collides directly with a particle $R$ of mass $m$ which is at rest on the plane. The coefficient of restitution between $Q$ and $R$ is $e$
(c) Calculate the range of values of $e$ for which there will be a second collision between $P$ and $Q$.

## 6678 <br> Edexcel GCE

Mechanics M2

## Advanced Subsidiary

Wednesday 21 May 2008 - Afternoon
Time: 1 hour 30 minutes

```
Materials required for examination
Mathematical Formulae (Green) Mathematical Formulae (Green)
```

Items included with question paper

Candidates may use any calculator allowed by the regulations of the Join Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them

## Instructions to Candidate

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature
Whenever a numerical value of $g$ is required, take $g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper
The total mark for this paper is 75 .

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may gain no credit.

1. A lorry of mass 2000 kg is moving down a straight road inclined at angle $\alpha$ to the horizontal where $\sin \alpha=\frac{1}{25}$. The resistance to motion is modelled as a constant force of magnitude 1600 N . The lorry is moving at a constant speed of $14 \mathrm{~m} \mathrm{~s}^{-1}$.

Find, in kW , the rate at which the lorry's engine is working.
2. A particle $A$ of mass $4 m$ is moving with speed $3 u$ in a straight line on a smooth horizontal table. The particle $A$ collides directly with a particle $B$ of mass $3 m$ moving with speed $2 u$ in the same direction as $A$. The coefficient of restitution between $A$ and $B$ is $e$. Immediately after the collision the speed of $B$ is $4 e u$.
(a) Show that $e=\frac{3}{4}$.
(b) Find the total kinetic energy lost in the collision.
3.


## Figure 1

A package of mass 3.5 kg is sliding down a ramp. The package is modelled as a particle and the ramp as a rough plane inclined at an angle of $20^{\circ}$ to the horizontal. The package slides down a line of greatest slope of the plane from a point $A$ to a point $B$, where $A B=14 \mathrm{~m}$. At $A$ the package has speed $12 \mathrm{~m} \mathrm{~s}^{-1}$ and at $B$ the package has speed $8 \mathrm{~m} \mathrm{~s}^{-1}$, as shown in Figure 1 .

Find
(a) the total energy lost by the package in travelling from $A$ to $B$,
(b) the coefficient of friction between the package and the ramp.
4. A particle $P$ of mass 0.5 kg is moving under the action of a single force $\mathbf{F}$ newtons. At time $t$ seconds,

$$
\mathbf{F}=(6 t-5) \mathbf{i}+\left(t^{2}-2 t\right) \mathbf{j}
$$

The velocity of $P$ at time $t$ seconds is $\mathbf{v} \mathrm{m} \mathrm{s}^{-1}$. When $t=0, \mathbf{v}=\mathbf{i}-4 \mathbf{j}$.
(a) Find $\mathbf{v}$ at time $t$ seconds.

When $t=3$, the particle $P$ receives an impulse $(-5 \mathbf{i}+12 \mathbf{j}) \mathrm{N} \mathrm{s}$.
(b) Find the speed of $P$ immediately after it receives the impulse.
5.


## Figure 2

A plank rests in equilibrium against a fixed horizontal pole. The plank is modelled as a uniform $\operatorname{rod} A B$ and the pole as a smooth horizontal peg perpendicular to the vertical plane containing $A B$. The rod has length $3 a$ and weight $W$ and rests on the peg at $C$, where $A C=2 a$. The end $A$ of the rod rests on rough horizontal ground and $A B$ makes an angle $\alpha$ with the ground, as shown in Figure 2.
(a) Show that the normal reaction on the rod at $A$ is $\frac{1}{4}\left(4-3 \cos ^{2} \alpha\right) W$.

Given that the rod is in limiting equilibrium and that $\cos \alpha=\frac{2}{3}$,
(b) find the coefficient of friction between the rod and the ground.
6.


Figure 3
Figure 3 shows a rectangular lamina $O A B C$. The coordinates of $O, A, B$ and $C$ are $(0,0)$, $(8,0),(8,5)$ and $(0,5)$ respectively. Particles of mass $k m, 5 m$ and $3 m$ are attached to the lamina at $A, B$ and $C$ respectively.

The $x$-coordinate of the centre of mass of the three particles without the lamina is 6.4.
(a) Show that $k=7$.

The lamina $O A B C$ is uniform and has mass 12 m .
(b) Find the coordinates of the centre of mass of the combined system consisting of the three particles and the lamina
(6)

The combined system is freely suspended from $O$ and hangs at rest.
(c) Find the angle between $O C$ and the horizontal.
7.


## Figure 4

A ball is thrown from a point $A$ at a target, which is on horizontal ground. The point $A$ is 12 m above the point $O$ on the ground. The ball is thrown from $A$ with speed $25 \mathrm{~m} \mathrm{~s}^{-1}$ at an angle of $30^{\circ}$ below the horizontal. The ball is modelled as a particle and the target as a point $T$. The distance $O T$ is 15 m . The ball misses the target and hits the ground at the point $B$, where $O T B$ is a straight line, as shown in Figure 4. Find
(a) the time taken by the ball to travel from $A$ to $B$,
(b) the distance $T B$.

The point $X$ is on the path of the ball vertically above $T$.
(c) Find the speed of the ball at $X$.

## 6678 <br> Edexcel GCE

## Mechanics M2

Advanced Level
Thursday 29 January 2009 - Morning
Time: 1 hour 30 minutes

## Materials required for examination Mathematical Formulae (Green) <br> $\frac{\text { Items included with question papers }}{}$

Candiates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

## Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature
Whenever a numerical value of $g$ is required, take $g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75 .

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner Answers without working may not gain full credit.

1. A car of mass 1500 kg is moving up a straight road, which is inclined at an angle $\theta$ to the horizontal, where $\sin \theta=\frac{1}{14}$. The resistance to the motion of the car from non-gravitational forces is constant and is modelled as a single constant force of magnitude 650 N . The car's engine is working at a rate of 30 kW .

Find the acceleration of the car at the instant when its speed is $15 \mathrm{~m} \mathrm{~s}^{-1}$.
2.


## Figure 1

Figure 1 shows a ladder $A B$, of mass 25 kg and length 4 m , resting in equilibrium with one end $A$ on rough horizontal ground and the other end $B$ against a smooth vertical wall. The ladder is in a vertical plane perpendicular to the wall. The coefficient of friction between the ladder and the ground is $\frac{11}{25}$. The ladder makes an angle $\beta$ with the ground. When Reece, who has mass 75 kg , stands at the point $C$ on the ladder, where $A C=2.8 \mathrm{~m}$, the ladder is on the point of slipping. The ladder is modelled as a uniform rod and Reece is modelled as a particle.
(a) Find the magnitude of the frictional force of the ground on the ladder.
(b) Find, to the nearest degree, the value of $\beta$.
(c) State how you have used the modelling assumption that Reece is a particle.
3. A block of mass 10 kg is pulled along a straight horizontal road by a constant horizontal force of magnitude 70 N in the direction of the road. The block moves in a straight line passing through two points $A$ and $B$ on the road, where $A B=50 \mathrm{~m}$. The block is modelled as a particle and the road is modelled as a rough plane. The coefficient of friction between the block and the road is $\frac{4}{7}$.
(a) Calculate the work done against friction in moving the block from $A$ to $B$.

The block passes through $A$ with a speed of $2 \mathrm{~m} \mathrm{~s}^{-1}$.
(b) Find the speed of the block at $B$.
4. A particle $P$ moves along the $x$-axis in a straight line so that, at time $t$ seconds, the velocity of $P$ is $v \mathrm{~m} \mathrm{~s}^{-1}$, where

$$
v= \begin{cases}10 t-2 t^{2}, & 0 \leq t \leq 6, \\ \frac{-432}{t^{2}}, & t>6 .\end{cases}
$$

At $t=0, P$ is at the origin $O$. Find the displacement of $P$ from $O$ when
(a) $t=6$,
(b) $t=10$.
5.


## Figure 2

A uniform lamina $A B C D$ is made by joining a uniform triangular lamina $A B D$ to a uniform semi-circular lamina $D B C$, of the same material, along the edge $B D$, as shown in Figure 2 Triangle $A B D$ is right-angled at $D$ and $A D=18 \mathrm{~cm}$. The semi-circle has diameter $B D$ and $B D=12 \mathrm{~cm}$.
(a) Show that, to 3 significant figures, the distance of the centre of mass of the lamina $A B C D$ from $A D$ is 4.69 cm .

Given that the centre of mass of a uniform semicircular lamina, radius $r$, is at a distance $\frac{4 r}{3 \pi}$ from the centre of the bounding diameter,
(b) find, in cm to 3 significant figures, the distance of the centre of mass of the lamina $A B C D$ from $B D$.

The lamina is freely suspended from $B$ and hangs in equilibrium.
(c) Find, to the nearest degree, the angle which $B D$ makes with the vertical
6.


## Figure 3

A cricket ball is hit from a point $A$ with velocity of $(p \mathbf{i}+q \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}$, at an angle $\alpha$ above the horizontal. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are respectively horizontal and vertically upwards. The point $A$ is 0.9 m vertically above the point $O$, which is on horizontal ground.

The ball takes 3 seconds to travel from $A$ to $B$, where $B$ is on the ground and $O B=57.6 \mathrm{~m}$, as shown in Figure 3. By modelling the motion of the cricket ball as that of a particle moving freely under gravity,
(a) find the value of $p$,
(b) show that $q=14.4$,
(c) find the initial speed of the cricket ball,
(d) find the exact value of $\tan \alpha$.
(e) Find the length of time for which the cricket ball is at least 4 m above the ground.
(f) State an additional physical factor which may be taken into account in a refinement of the above model to make it more realistic
7. A particle $P$ of mass $3 m$ is moving in a straight line with speed $2 u$ on a smooth horizontal table. It collides directly with another particle $Q$ of mass $2 m$ which is moving with speed $u$ in the opposite direction to $P$. The coefficient of restitution between $P$ and $Q$ is $e$.
(a) Show that the speed of $Q$ immediately after the collision is $\frac{1}{5}(9 e+4) u$.

The speed of $P$ immediately after the collision is $\frac{1}{2} u$.
(b) Show that $e=\frac{1}{4}$.

The collision between $P$ and $Q$ takes place at the point $A$. After the collision $Q$ hits a smooth fixed vertical wall which is at right-angles to the direction of motion of $Q$. The distance from $A$ to the wall is $d$.
(c) Show that $P$ is a distance $\frac{3}{5} d$ from the wall at the instant when $Q$ hits the wall.

Particle $Q$ rebounds from the wall and moves so as to collide directly with particle $P$ at the point $B$. Given that the coefficient of restitution between $Q$ and the wall is $\frac{1}{5}$,
(d) find, in terms of $d$, the distance of the point $B$ from the wall.

## 6678 <br> Edexcel GCE

## Mechanics M2

## Advanced

Friday 22 May 2009 - Morning
Time: $\mathbf{1}$ hour 30 minutes

```
Materials required for examination athematical Formulae (Orange or Green)
```

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

## Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature
Whenever a numerical value of $g$ is required, take $g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information for Candidate

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper.
The total mark for this paper is 75 .

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner Answers without working may not gain full credit.

1. A particle of mass 0.25 kg is moving with velocity $(3 \mathbf{i}+7 \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}$ when it receives the impulse $(5 \mathbf{i}-3 \mathbf{j}) \mathrm{N} \mathrm{s}$.

Find the speed of the particle immediately after the impulse.
2. At time $t=0$ a particle $P$ leaves the origin $O$ and moves along the $x$-axis. At time $t$ seconds the velocity of $P$ is $v \mathrm{~m} \mathrm{~s}^{-1}$, where

$$
v=8 t-t^{2} .
$$

(a) Find the maximum value of $v$.
(b) Find the time taken for $P$ to return to $O$
3. A truck of mass of 300 kg moves along a straight horizontal road with a constant speed of $10 \mathrm{~m} \mathrm{~s}^{-1}$. The resistance to motion of the truck has magnitude 120 N .
(a) Find the rate at which the engine of the truck is working.

On another occasion the truck moves at a constant speed up a hill inclined at $\theta$ to the horizontal, where $\sin \theta=\frac{1}{14}$. The resistance to motion of the truck from non-gravitational forces remains of magnitude 120 N . The rate at which the engine works is the same as in part (a).
(b) Find the speed of the truck.
4.


## Figure 1

A uniform rod $A B$, of length 1.5 m and mass 3 kg , is smoothly hinged to a vertical wall at $A$. The rod is held in equilibrium in a horizontal position by a light strut $C D$ as shown in Figure 1. The rod and the strut lie in the same vertical plane, which is perpendicular to the wall. The end $C$ of the strut is freely jointed to the wall at a point 0.5 m vertically below $A$. The end $D$ is freely joined to the rod so that $A D$ is 0.5 m .
a) Find the thrust in $C D$.
(b) Find the magnitude and direction of the force exerted on the $\operatorname{rod} A B$ at $A$.


## Figure 2

A shop sign $A B C D E F G$ is modelled as a uniform lamina, as illustrated in Figure 2. $A B C D$ is a rectangle with $B C=120 \mathrm{~cm}$ and $D C=90 \mathrm{~cm}$. The shape $E F G$ is an isosceles triangle with $E G=60 \mathrm{~cm}$ and height 60 cm . The mid-point of $A D$ and the mid-point of $E G$ coincide.
(a) Find the distance of the centre of mass of the sign from the side $A D$.

The sign is freely suspended from $A$ and hangs at rest.
(b) Find the size of the angle between $A B$ and the vertical.
6.


O

## Figure 3

A child playing cricket on horizontal ground hits the ball towards a fence 10 m away. The ball moves in a vertical plane which is perpendicular to the fence. The ball just passes over the top of the fence, which is 2 m above the ground, as shown in Figure 3.

The ball is modelled as a particle projected with initial speed $u \mathrm{~m} \mathrm{~s}^{-1}$ from point $O$ on the ground at an angle $\alpha$ to the ground.
(a) By writing down expressions for the horizontal and vertical distances, from $O$ of the ball $t$ seconds after it was hit, show that

$$
\begin{equation*}
2=10 \tan \alpha-\frac{50 g}{u^{2} \cos ^{2} \alpha} . \tag{6}
\end{equation*}
$$

Given that $\alpha=45^{\circ}$,
(b) find the speed of the ball as it passes over the fence.
7.


Figure 4
A particle $P$ of mass 2 kg is projected up a rough plane with initial speed $14 \mathrm{~m} \mathrm{~s}^{-1}$, from a point $X$ on the plane, as shown in Figure 4. The particle moves up the plane along the line of greatest slope through $X$ and comes to instantaneous rest at the point $Y$. The plane is inclined at an angle $\alpha$ to the horizontal, where $\tan \alpha=\frac{7}{24}$. The coefficient of friction between the particle and the plane is $\frac{1}{8}$.
(a) Use the work-energy principle to show that $X Y=25 \mathrm{~m}$.

After reaching $Y$, the particle $P$ slides back down the plane.
(b) Find the speed of $P$ as it passes through $X$.
8. Particles $A, B$ and $C$ of masses $4 m, 3 m$ and $m$ respectively, lie at rest in a straight line on a smooth horizontal plane with $B$ between $A$ and $C$. Particles $A$ and $B$ are projected towards each other with speeds $u \mathrm{~m} \mathrm{~s}^{-1}$ and $v \mathrm{~m} \mathrm{~s}^{-1}$ respectively, and collide directly. As a result of the collision, $A$ is brought to rest and $B$ rebounds with speed $k v \mathrm{~m} \mathrm{~s}^{-1}$. The coefficient of restitution between $A$ and $B$ is $\frac{3}{4}$.
(a) Show that $u=3 v$.
(b) Find the value of $k$.

Immediately after the collision between $A$ and $B$, particle $C$ is projected with speed $2 v \mathrm{~m} \mathrm{~s}^{-1}$ towards $B$ so that $B$ and $C$ collide directly.
(c) Show that there is no further collision between $A$ and $B$

## 6678 <br> Edexcel GCE

## Mechanics M2

## Advanced Level

Friday 29 January 2010 - Afternoon
Time: $\mathbf{1}$ hour 30 minutes

## Materials required for examination Mathematical Formulae (Pink or Green) <br> Items included with question papers

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

## Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature
Whenever a numerical value of $g$ is required, take $g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper.
The total mark for this paper is 75

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner Answers without working may not gain full credit.

1. A particle $P$ moves along the $x$-axis. At time $t$ seconds the velocity of $P$ is $v \mathrm{~m} \mathrm{~s}^{-1}$ in the positive $x$-direction, where $v=3 t^{2}-4 t+3$. When $t=0, P$ is at the origin $O$. Find the distance of $P$ from $O$ when $P$ is moving with minimum velocity.
2. Two particles, $P$, of mass $2 m$, and $Q$, of mass $m$, are moving along the same straight line on a smooth horizontal plane. They are moving in opposite directions towards each other and collide. Immediately before the collision the speed of $P$ is $2 u$ and the speed of $Q$ is $u$. The coefficient of restitution between the particles is $e$, where $e<1$. Find, in terms of $u$ and $e$,
(i) the speed of $P$ immediately after the collision,
(ii) the speed of $Q$ immediately after the collision.
3. A particle of mass 0.5 kg is projected vertically upwards from ground level with a speed of $20 \mathrm{~m} \mathrm{~s}^{-1}$. It comes to instantaneous rest at a height of 10 m above the ground. As the particle moves it is subject to air resistance of constant magnitude $R$ newtons. Using the workenergy principle, or otherwise, find the value of $R$.
4. 



## Figure 1

The points $A, B$ and $C$ lie in a horizontal plane. A batsman strikes a ball of mass 0.25 kg . Immediately before being struck, the ball is moving along the horizontal line $A B$ with speed $30 \mathrm{~m} \mathrm{~s}^{-1}$. Immediately after being struck, the ball moves along the horizontal line $B C$ with speed $40 \mathrm{~m} \mathrm{~s}^{-1}$. The line $B C$ makes an angle of $60^{\circ}$ with the original direction of motion $A B$, as shown in Figure 1.

Find, to 3 significant figures,
(i) the magnitude of the impulse given to the ball,
(ii) the size of the angle that the direction of this impulse makes with the original direction of motion $A B$.
5. A cyclist and her bicycle have a total mass of 70 kg . She cycles along a straight horizontal road with constant speed $3.5 \mathrm{~m} \mathrm{~s}^{-1}$. She is working at a constant rate of 490 W .
(a) Find the magnitude of the resistance to motion.

The cyclist now cycles down a straight road which is inclined at an angle $\theta$ to the horizontal, where $\sin \theta=\frac{1}{14}$, at a constant speed $U \mathrm{~m} \mathrm{~s}^{-1}$. The magnitude of the non-gravitational resistance to motion is modelled as $40 U$ newtons. She is now working at a constant rate of 24 W .
(b) Find the value of $U$.
6.


## Figure 2

A uniform rod $A B$, of mass 20 kg and length 4 m , rests with one end $A$ on rough horizontal ground. The rod is held in limiting equilibrium at an angle $\alpha$ to the horizontal, where $\tan \alpha=\frac{3}{4}$, by a force acting at $B$, as shown in Figure 2. The line of action of this force lies in the vertical plane which contains the rod. The coefficient of friction between the ground and the rod is 0.5 .

Find the magnitude of the normal reaction of the ground on the $\operatorname{rod}$ at $A$.
7. [The centre of mass of a semi-circular lamina of radius $r$ is $\frac{4 r}{3 \pi}$ from the centre.]


## Figure 3

A template $T$ consists of a uniform plane lamina $P Q R O S$, as shown in Figure 3. The lamina is bounded by two semicircles, with diameters $S O$ and $O R$, and by the sides $S P, P Q$ and $Q R$ of the rectangle $P Q R S$. The point $O$ is the mid-point of $S R, P Q=12 \mathrm{~cm}$ and $Q R=2 x \mathrm{~cm}$.
(a) Show that the centre of mass of $T$ is a distance $\frac{4\left|2 x^{2}-3\right|}{8 x+3 \pi} \mathrm{~cm}$ from $S R$.

The template $T$ is freely suspended from the point $P$ and hangs in equilibrium.
Given that $x=2$ and that $\theta$ is the angle that $P Q$ makes with the horizontal,
(b) show that $\tan \theta=\frac{48+9 \pi}{22+6 \pi}$.
8. [In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in a horizontal and upward vertical direction respectively.]

A particle $P$ is projected from a fixed point $O$ on horizontal ground with velocity $u(\mathbf{i}+c \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}$, where $c$ and $u$ are positive constants. The particle moves freely under gravity until it strikes the ground at $A$, where it immediately comes to rest. Relative to $O$, the position vector of a point on the path of $P$ is $(x \mathbf{i}+y \mathbf{j}) \mathrm{m}$.
(a) Show that

$$
\begin{equation*}
y=c x-\frac{4.9 x^{2}}{u^{2}} \tag{5}
\end{equation*}
$$

Given that $u=7, O A=R \mathrm{~m}$ and the maximum vertical height of $P$ above the ground is $H \mathrm{~m}$,
(b) using the result in part (a), or otherwise, find, in terms of $c$,
(i) $R$
(ii) $H$

Given also that when $P$ is at the point $Q$, the velocity of $P$ is at right angles to its initial velocity,
(c) find, in terms of $c$, the value of $x$ at $Q$

## TOTAL FOR PAPER: 75 MARKS

 END
## 6678 <br> Edexcel GCE

## Mechanics M2

## Advanced

## Friday 11 June 2010 - Afternoon

Time: 1 hour 30 minutes

## Materials required for examinatio <br> Mathematical Formulae (Pink)

Items included with question papers

Candidates may use any calculator allowed by the regulations of the Joint Candidates may use any calculator allowed by the regutains of he Joint algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

## Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature
Whenever a numerical value of $g$ is required, take $g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper
The total mark for this paper is 75 .

## Advice to Candidate

You must ensure that your answers to parts of questions are clearly labelled
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. A particle $P$ moves on the $x$-axis. The acceleration of $P$ at time $t$ seconds, $t \geq 0$, is $(3 t+5) \mathrm{m} \mathrm{s}^{-2}$ in the positive $x$-direction. When $t=0$, the velocity of $P$ is $2 \mathrm{~m} \mathrm{~s}^{-1}$ in the positive $x$-direction. When $t=T$, the velocity of $P$ is $6 \mathrm{~m} \mathrm{~s}^{-1}$ in the positive $x$-direction.

Find the value of $T$.
2. A particle $P$ of mass 0.6 kg is released from rest and slides down a line of greatest slope of a rough plane. The plane is inclined at $30^{\circ}$ to the horizontal. When $P$ has moved 12 m , its speed is $4 \mathrm{~m} \mathrm{~s}^{-1}$. Given that friction is the only non-gravitational resistive force acting on $P$, find
(a) the work done against friction as the speed of $P$ increases from $0 \mathrm{~m} \mathrm{~s}^{-1}$ to $4 \mathrm{~m} \mathrm{~s}^{-1}$,
(b) the coefficient of friction between the particle and the plane
3.


Figure 1
A triangular frame is formed by cutting a uniform rod into 3 pieces which are then joined to form a triangle $A B C$, where $A B=A C=10 \mathrm{~cm}$ and $B C=12 \mathrm{~cm}$, as shown in Figure 1
(a) Find the distance of the centre of mass of the frame from $B C$.

The frame has total mass $M$. A particle of mass $M$ is attached to the frame at the mid-point of $B C$. The frame is then freely suspended from $B$ and hangs in equilibrium.
(b) Find the size of the angle between $B C$ and the vertical.
4. A car of mass 750 kg is moving up a straight road inclined at an angle $\theta$ to the horizontal, where $\sin \theta=\frac{1}{15}$. The resistance to motion of the car from non-gravitational forces has constant magnitude $R$ newtons. The power developed by the car's engine is 15 kW and the car is moving at a constant speed of $20 \mathrm{~m} \mathrm{~s}^{-1}$.
(a) Show that $R=260$.

The power developed by the car's engine is now increased to 18 kW . The magnitude of the resistance to motion from non-gravitational forces remains at 260 N . At the instant when the car is moving up the road at $20 \mathrm{~m} \mathrm{~s}^{-1}$ the car's acceleration is $a \mathrm{~m} \mathrm{~s}^{-2}$.
(b) Find the value of $a$.
5. [In this question $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors in a horizontal plane.]

A ball of mass 0.5 kg is moving with velocity $(10 \mathbf{i}+24 \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}$ when it is struck by a bat. Immediately after the impact the ball is moving with velocity $20 \mathrm{i} \mathrm{m} \mathrm{s}^{-1}$.

Find
(a) the magnitude of the impulse of the bat on the ball,
(b) the size of the angle between the vector $\mathbf{i}$ and the impulse exerted by the bat on the ball,
(c) the kinetic energy lost by the ball in the impact.
6.


## Figure 2

Figure 2 shows a uniform rod $A B$ of mass $m$ and length $4 a$. The end $A$ of the rod is freely hinged to a point on a vertical wall. A particle of mass $m$ is attached to the rod at $B$. One end of a light inextensible string is attached to the rod at $C$, where $A C=3 a$. The other end of the string is attached to the wall at $D$, where $A D=2 a$ and $D$ is vertically above $A$. The rod rests horizontally in equilibrium in a vertical plane perpendicular to the wall and the tension in the string is $T$.
(a) Show that $T=m g \sqrt{ } 13$

The particle of mass $m$ at $B$ is removed from the rod and replaced by a particle of mass $M$ which is attached to the rod at $B$. The string breaks if the tension exceeds $2 \mathrm{mg} \sqrt{ } 13$. Given that the string does not break,
(b) show that $M \leq \frac{5}{2} m$.

7


Figure 3
A ball is projected with speed $40 \mathrm{~m} \mathrm{~s}^{-1}$ from a point $P$ on a cliff above horizontal ground. The point $O$ on the ground is vertically below $P$ and $O P$ is 36 m . The ball is projected at an angle $\theta^{\circ}$ to the horizontal. The point $Q$ is the highest point of the path of the ball and is 12 m above the level of $P$. The ball moves freely under gravity and hits the ground at the point $R$, as shown in Figure 3. Find
(a) the value of $\theta$,
(b) the distance $O R$,
(c) the speed of the ball as it hits the ground at $R$
8. A small ball $A$ of mass $3 m$ is moving with speed $u$ in a straight line on a smooth horizontal table. The ball collides directly with another small ball $B$ of mass $m$ moving with speed $u$ towards $A$ along the same straight line. The coefficient of restitution between $A$ and $B$ is $\frac{1}{2}$. The balls have the same radius and can be modelled as particles.
(a) Find
(i) the speed of $A$ immediately after the collision,
(ii) the speed of $B$ immediately after the collision.

After the collision $B$ hits a smooth vertical wall which is perpendicular to the direction of motion of $B$. The coefficient of restitution between $B$ and the wall is $\frac{2}{5}$.
(b) Find the speed of $B$ immediately after hitting the wall.

The first collision between $A$ and $B$ occurred at a distance $4 a$ from the wall. The balls collide again $T$ seconds after the first collision.
(c) Show that $T=\frac{112 a}{15 u}$.

TOTAL FOR PAPER: 75 MARKS

## 6678 <br> Edexcel GCE

Mechanics M2

## Advanced Level

Friday 28 January 2011 - Morning
Time: 1 hour 30 minutes

```
Materials required for examination
Mathematical Formulae (Pink)
\(\frac{\text { Items included with question paper }}{\text { Nil }}\) Mathematical Formulae (Pink)
```

Candidates may use any calculator allowed by the regulations of the Join Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

## Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), you centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature
Whenever a numerical value of $g$ is required, take $g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper
The total mark for this paper is 75 .

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled
You must show sufficient working to make your methods clear to the Examiner. Answers without working may not gain full credit.

1. A cyclist starts from rest and moves along a straight horizontal road. The combined mass of the cyclist and his cycle is 120 kg . The resistance to motion is modelled as a constant force of magnitude 32 N . The rate at which the cyclist works is 384 W . The cyclist accelerates until he reaches a constant speed of $v \mathrm{~m} \mathrm{~s}^{-1}$.

Find
(a) the value of $v$,
(b) the acceleration of the cyclist at the instant when the speed is $9 \mathrm{~m} \mathrm{~s}^{-1}$.
2. A particle of mass 2 kg is moving with velocity $(5 \mathbf{i}+\mathbf{j}) \mathrm{m} \mathrm{s}^{-1}$ when it receives an impulse of $(-6 \mathbf{i}+8 \mathbf{j}) \mathrm{N}$ s. Find the kinetic energy of the particle immediately after receiving the impulse.
3. A particle moves along the $x$-axis. At time $t=0$ the particle passes through the origin with speed $8 \mathrm{~m} \mathrm{~s}^{-1}$ in the positive $x$-direction. The acceleration of the particle at time $t$ seconds, $t \geq 0$, is $\left(4 t^{3}-12 t\right) \mathrm{m} \mathrm{s}^{-2}$ in the positive $x$-direction.

Find
(a) the velocity of the particle at time $t$ seconds,
(b) the displacement of the particle from the origin at time $t$ seconds,
(c) the values of $t$ at which the particle is instantaneously at rest.
4.
(a) find the work done in dragging the box from $A$ to $B$. principle, or otherwise,
(b) find the speed of the box as it reaches $A$.


## Figure 1

A box of mass 30 kg is held at rest at point $A$ on a rough inclined plane. The plane is inclined at $20^{\circ}$ to the horizontal. Point $B$ is 50 m from $A$ up a line of greatest slope of the plane, as shown in Figure 1. The box is dragged from $A$ to $B$ by a force acting parallel to $A B$ and then held at rest at $B$. The coefficient of friction between the box and the plane is $\frac{1}{4}$. Friction is the only non-gravitational resistive force acting on the box. Modelling the box as a particle,

The box is released from rest at the point $B$ and slides down the slope. Using the work-energy
5.



## Figure 2

The uniform L-shaped lamina $A B C D E F$, shown in Figure 2, has sides $A B$ and $F E$ parallel, and sides $B C$ and $E D$ parallel. The pairs of parallel sides are 9 cm apart. The points $A, F, D$ and $C$ lie on a straight line.
$A B=B C=36 \mathrm{~cm}, F E=E D=18 \mathrm{~cm}$.
$\angle A B C=\angle F E D=90^{\circ}$, and $\angle B C D=\angle E D F=\angle E F D=\angle B A C=45^{\circ}$.
(a) Find the distance of the centre of mass of the lamina from
(i) side $A B$,
(ii) side $B C$.

The lamina is freely suspended from $A$ and hangs in equilibrium.
(b) Find, to the nearest degree, the size of the angle between $A B$ and the vertical.
6. [In this question, the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are in a vertical plane, $\mathbf{i}$ being horizontal and $\mathbf{j}$ being vertically upwards.]


## Figure 3

At time $t=0$, a particle $P$ is projected from the point $A$ which has position vector $10 \mathbf{j}$ metres with respect to a fixed origin $O$ at ground level. The ground is horizontal. The velocity of projection of $P$ is $(3 \mathbf{i}+5 \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}$, as shown in Figure 3. The particle moves freely under gravity and reaches the ground after $T$ seconds
(a) For $0 \leq t \leq T$, show that, with respect to $O$, the position vector, $\mathbf{r}$ metres, of $P$ at time $t$ seconds is given by

$$
\begin{equation*}
\mathbf{r}=3 t \mathbf{i}+\left(10+5 t-4.9 t^{2}\right) \mathbf{j} \tag{3}
\end{equation*}
$$

(b) Find the value of $T$.
(c) Find the velocity of $P$ at time $t$ seconds $(0 \leq t \leq T)$.

When $P$ is at the point $B$, the direction of motion of $P$ is $45^{\circ}$ below the horizontal.
(d) Find the time taken for $P$ to move from $A$ to $B$.
(e) Find the speed of $P$ as it passes through $B$.
7.


Figure 4
A uniform plank $A B$, of weight 100 N and length 4 m , rests in equilibrium with the end $A$ on rough horizontal ground. The plank rests on a smooth cylindrical drum. The drum is fixed to he ground and cannot move. The point of contact between the plank and the drum is $C$, where $A C=3 \mathrm{~m}$, as shown in Figure 4. The plank is resting in a vertical plane which is perpendicular to the axis of the drum, at an angle $\alpha$ to the horizontal, where $\sin \alpha=\frac{1}{3}$. The coefficient of friction between the plank and the ground is $\mu$.

Modelling the plank as a rod, find the least possible value of $\mu$.
8. A particle $P$ of mass $m \mathrm{~kg}$ is moving with speed $6 \mathrm{~m} \mathrm{~s}^{-1}$ in a straight line on a smooth horizontal floor. The particle strikes a fixed smooth vertical wall at right angles and rebounds The kinetic energy lost in the impact is 64 J . The coefficient of restitution between $P$ and the wall is $\frac{1}{3}$.
(a) Show that $m=4$

After rebounding from the wall, $P$ collides directly with a particle $Q$ which is moving towards $P$ with speed $3 \mathrm{~m} \mathrm{~s}^{-1}$. The mass of $Q$ is 2 kg and the coefficient of restitution between $P$ and $Q$ is $\frac{1}{3}$.
(b) Show that there will be a second collision between $P$ and the wall.

## TOTAL FOR PAPER: 75 MARKS

END

## 6677/01 <br> Edexcel GCE

## Mechanics M2

Advanced Level
Monday 13 June 2011 - Morning
Time: $\mathbf{1}$ hour 30 minutes

```
Materials required for examinatio
Mathematical Formulae (Pink)

Candates may use any calcular allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them

\section*{Instructions to Candidates}

In the boxes on the answer book, write the name of the examining body (Edexcel), you centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature
Whenever a numerical value of \(g\) is required, take \(g=9.8 \mathrm{~m} \mathrm{~s}^{-2}\)
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

\section*{Information for Candidates}

A booklet 'Mathematical Formulae and Statistical Tables' is provided
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper
The total mark for this paper is 75

\section*{Advice to Candidates}

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may not gain full credit.
1. A car of mass 1000 kg moves with constant speed \(V \mathrm{~m} \mathrm{~s}^{-1}\) up a straight road inclined at an angle \(\theta\) to the horizontal, where \(\sin \theta=\frac{1}{30}\). The engine of the car is working at a rate of 12 kW . The resistance to motion from non-gravitational forces has magnitude 500 N .

Find the value of \(V\).
2. A particle \(P\) of mass \(m\) is moving in a straight line on a smooth horizontal surface with speed \(4 u\). The particle \(P\) collides directly with a particle \(Q\) of mass \(3 m\) which is at rest on the surface. The coefficient of restitution between \(P\) and \(Q\) is \(e\). The direction of motion of \(P\) is reversed by the collision.

Show that \(e>\frac{1}{3}\).
3. A ball of mass 0.5 kg is moving with velocity \(12 \mathrm{i} \mathrm{m} \mathrm{s}^{-1}\) when it is struck by a bat. The impulse received by the ball is \((-4 \mathbf{i}+7 \mathbf{j}) \mathrm{N}\) s. By modelling the ball as a particle, find
(a) the speed of the ball immediately after the impact,
(b) the angle, in degrees, between the velocity of the ball immediately after the impact and the vector \(\mathbf{i}\),
(c) the kinetic energy gained by the ball as a result of the impact.


Figure 1
Figure 1 shows a uniform lamina \(A B C D E\) such that \(A B D E\) is a rectangle, \(B C=C D, A B=4 a\) and \(A E=2 a\). The point \(F\) is the midpoint of \(B D\) and \(F C=a\).
(a) Find, in terms of \(a\), the distance of the centre of mass of the lamina from \(A E\)

The lamina is freely suspended from \(A\) and hangs in equilibrium.
(b) Find the angle between \(A B\) and the downward vertical.

P38162A
5.


\section*{Figure 2}

A particle \(P\) of mass 0.5 kg is projected from a point \(A\) up a line of greatest slope \(A B\) of a fixed plane. The plane is inclined at \(30^{\circ}\) to the horizontal and \(A B=2 \mathrm{~m}\) with \(B\) above \(A\), as shown in Figure 2. The particle \(P\) passes through \(B\) with speed \(5 \mathrm{~m} \mathrm{~s}^{-1}\). The plane is smooth from \(A\) to \(B\).
(a) Find the speed of projection.

The particle \(P\) comes to instantaneous rest at the point \(C\) on the plane, where \(C\) is above \(B\) and \(B C=1.5 \mathrm{~m}\). From \(B\) to \(C\) the plane is rough and the coefficient of friction between \(P\) and the plane is \(\mu\).

By using the work-energy principle,
(b) find the value of \(\mu\).
6. A particle \(P\) moves on the \(x\)-axis. The acceleration of \(P\) at time \(t\) seconds is \((t-4) \mathrm{m} \mathrm{s}^{-2}\) in the positive \(x\)-direction. The velocity of \(P\) at time \(t\) seconds is \(v \mathrm{~m} \mathrm{~s}^{-1}\). When \(t=0, v=6\).

Find
(a) \(v\) in terms of \(t\),
(b) the values of \(t\) when \(P\) is instantaneously at rest,
(c) the distance between the two points at which \(P\) is instantaneously at rest.

 .

都
-
7.
\(\square\)


\section*{Figure 3}

A uniform \(\operatorname{rod} A B\), of mass \(3 m\) and length \(4 a\), is held in a horizontal position with the end \(A\) against a rough vertical wall. One end of a light inextensible string \(B D\) is attached to the rod at \(B\) and the other end of the string is attached to the wall at the point \(D\) vertically above \(A\), where \(A D=3 a\). A particle of mass \(3 m\) is attached to the rod at \(C\), where \(A C=x\). The rod is in equilibrium in a vertical plane perpendicular to the wall as shown in Figure 3. The tension in the string is \(\frac{25}{4} \mathrm{mg}\).

Show that
(a) \(x=3 a\),
(b) the horizontal component of the force exerted by the wall on the rod has magit

The coefficient of friction between the wall and the rod is \(\mu\). Given that the rod is about to slip,
(c) find the value of \(\mu\).
8. A particle is projected from a point \(O\) with speed \(u\) at an angle of elevation \(\alpha\) above the horizontal and moves freely under gravity. When the particle has moved a horizontal distance \(x\), its height above \(O\) is \(y\).
(a) Show that
\[
\begin{equation*}
y=x \tan \alpha-\frac{g x^{2}}{2 u^{2} \cos ^{2} \alpha} . \tag{4}
\end{equation*}
\]

A girl throws a ball from a point \(A\) at the top of a cliff. The point \(A\) is 8 m above a horizontal beach. The ball is projected with speed \(7 \mathrm{~m} \mathrm{~s}^{-1}\) at an angle of elevation of \(45^{\circ}\). By modelling the ball as a particle moving freely under gravity,
(b) find the horizontal distance of the ball from \(A\) when the ball is 1 m above the beach.

A boy is standing on the beach at the point \(B\) vertically below \(A\). He starts to run in a straight line with speed \(v \mathrm{~m} \mathrm{~s}^{-1}\), leaving \(B 0.4\) seconds after the ball is thrown.

He catches the ball when it is 1 m above the beach.
(c) Find the value of \(v\).

\section*{6678 \\ Edexcel GCE}

\section*{Mechanics M2}

\section*{Advanced Level}

Friday 27 January 2012 - Morning
Time: 1 hour 30 minutes
```

Materials required for examination
Mathematical Formulae (Pink) Mathematical Formulae (Pink)

```
\(\frac{\text { Items included with question papers }}{\text { Nil }}\)

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

\section*{Instructions to Candidates}

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature
Whenever a numerical value of \(g\) is required, take \(g=9.8 \mathrm{~m} \mathrm{~s}^{-2}\).
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

\section*{Information for Candidates}

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75 .

\section*{Advice to Candidates}

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner Answers without working may not gain full credit.
1. A tennis ball of mass 0.1 kg is hit by a racquet. Immediately before being hit, the ball has velocity \(30 \mathbf{i} \mathrm{~m} \mathrm{~s}^{-1}\). The racquet exerts an impulse of \((-2 \mathbf{i}-4 \mathbf{j}) \mathrm{N} \mathrm{s}\) on the ball. By modelling the ball as a particle, find the velocity of the ball immediately after being hit
2. A particle \(P\) is moving in a plane. At time \(t\) seconds, \(P\) is moving with velocity \(\mathbf{v} \mathrm{m} \mathrm{s}^{-1}\), where \(\mathbf{v}=2 t \mathbf{i}-3 t^{2} \mathbf{j}\)

Find
(a) the speed of \(P\) when \(t=4\)
(b) the acceleration of \(P\) when \(t=4\)

Given that \(P\) is at the point with position vector \((-4 \mathbf{i}+\mathbf{j}) \mathrm{m}\) when \(t=1\)
(c) find the position vector of \(P\) when \(t=4\)
3. A cyclist and her cycle have a combined mass of 75 kg . The cyclist is cycling up a straight road inclined at \(5^{\circ}\) to the horizontal. The resistance to the motion of the cyclist from non-gravitational forces is modelled as a constant force of magnitude 20 N . At the instant when the cyclist has a speed of \(12 \mathrm{~m} \mathrm{~s}^{-1}\), she is decelerating at \(0.2 \mathrm{~m} \mathrm{~s}^{-2}\).
(a) Find the rate at which the cyclist is working at this instant.

When the cyclist passes the point \(A\) her speed is \(8 \mathrm{~m} \mathrm{~s}^{-1}\). At \(A\) she stops working but does not apply the brakes. She comes to rest at the point \(B\).

The resistance to motion from non-gravitational forces is again modelled as a constant force of magnitude 20 N
(b) Use the work-energy principle to find the distance \(A B\).
4.


Figure 1
The trapezium \(A B C D\) is a uniform lamina with \(A B=4 \mathrm{~m}\) and \(B C=C D=D A=2 \mathrm{~m}\), as shown in Figure 1.
(a) Show that the centre of mass of the lamina is \(\frac{4 \sqrt{ } 3}{9} \mathrm{~m}\) from \(A B\).

The lamina is freely suspended from \(D\) and hangs in equilibrium.
(b) Find the angle between \(D C\) and the vertical through \(D\).
5.


\section*{Figure 2}

A uniform \(\operatorname{rod} A B\) has mass 4 kg and length 1.4 m . The end \(A\) is resting on rough horizontal ground. A light string \(B C\) has one end attached to \(B\) and the other end attached to a fixed point \(C\). The string is perpendicular to the rod and lies in the same vertical plane as the rod. The rod is in equilibrium, inclined at \(20^{\circ}\) to the ground, as shown in Figure 2.
(a) Find the tension in the string.

Given that the rod is about to slip,
(b) find the coefficient of friction between the rod and the ground.
6. Three identical particles, \(A, B\) and \(C\), lie at rest in a straight line on a smooth horizontal table with \(B\) between \(A\) and \(C\). The mass of each particle is \(m\). Particle \(A\) is projected towards \(B\) with speed \(u\) and collides directly with \(B\). The coefficient of restitution between each pair of particles is \(\frac{2}{3}\).
(a) Find, in terms of \(u\),
(i) the speed of \(A\) after this collision,
(ii) the speed of \(B\) after this collision.
(b) Show that the kinetic energy lost in this collision is \(\frac{5}{36} m u^{2}\).

After the collision between \(A\) and \(B\), particle \(B\) collides directly with \(C\).
(c) Find, in terms of \(u\), the speed of \(C\) immediately after this collision between \(B\) and \(C\).
7. [In this question, the unit vectors \(\mathbf{i}\) and \(\mathbf{j}\) are horizontal and vertical respectively.]


\section*{Figure 3}

The point \(O\) is a fixed point on a horizontal plane. A ball is projected from \(O\) with velocity \((6 \mathbf{i}+12 \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}\), and passes through the point \(A\) at time \(t\) seconds after projection. The point \(B\) is on the horizontal plane vertically below \(A\), as shown in Figure 3. It is given that \(O B=2 A B\).

Find
(a) the value of \(t\),
(b) the speed, \(V \mathrm{~m} \mathrm{~s}^{-1}\), of the ball at the instant when it passes through \(A\).

At another point \(C\) on the path the speed of the ball is also \(V \mathrm{~m} \mathrm{~s}^{-1}\).
(c) Find the time taken for the ball to travel from \(O\) to \(C\).

\section*{6677/01 \\ Edexcel GCE}

\section*{Mechanics M2}

\section*{Advanced Level}

Thursday 31 May 2012 - Morning
Time: 1 hour 30 minutes
```

Materials required for examinatio
Mathematical Formulae (Pink)
$\frac{\text { Items included with question paper }}{}$ Mathematical Formulae (Pink)

```

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

\section*{Instructions to Candidates}

In the boxes on the answer book, write the name of the examining body (Edexcel), you centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature
Whenever a numerical value of \(g\) is required, take \(g=9.8 \mathrm{~m} \mathrm{~s}^{-2}\).
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

\section*{Information for Candidates}

A booklet 'Mathematical Formulae and Statistical Tables' is provided
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper
The total mark for this paper is 75 .

\section*{Advice to Candidates}

You must ensure that your answers to parts of questions are clearly labelled
You must show sufficient working to make your methods clear to the Examiner. Answers without working may not gain full credit.
1. [In this question \(\mathbf{i}\) and \(\mathbf{j}\) are perpendicular unit vectors in a horizontal plane.]

A particle \(P\) moves in such a way that its velocity \(\mathbf{v} \mathrm{m} \mathrm{s}^{-1}\) at time \(t\) seconds is given by
\[
\mathbf{v}=\left(3 t^{2}-1\right) \mathbf{i}+\left(4 t-t^{2}\right) \mathbf{j} .
\]
(a) Find the magnitude of the acceleration of \(P\) when \(t=1\).

Given that, when \(t=0\), the position vector of \(P\) is \(\mathbf{i}\) metres,
(b) find the position vector of \(P\) when \(t=3\).
2. A particle \(P\) of mass \(3 m\) is moving with speed \(2 u\) in a straight line on a smooth horizontal plane. The particle \(P\) collides directly with a particle \(Q\) of mass \(4 m\) moving on the plane with speed \(u\) in the opposite direction to \(P\). The coefficient of restitution between \(P\) and \(Q\) is \(e\).
a) Find the speed of \(Q\) immediately after the collision.

Given that the direction of motion of \(P\) is reversed by the collision,
(b) find the range of possible values of \(e\).


Figure 1
A uniform \(\operatorname{rod} A B\), of mass 5 kg and length 4 m , has its end \(A\) smoothly hinged at a fixed point. The rod is held in equilibrium at an angle of \(25^{\circ}\) above the horizontal by a force of magnitude \(F\) newtons applied to its end \(B\). The force acts in the vertical plane containing the rod and in a direction which makes an angle of \(40^{\circ}\) with the rod, as shown in Figure 1.
(a) Find the value of \(F\).
(b) Find the magnitude and direction of the vertical component of the force acting on the rod at \(A\)


\section*{Figure 2}

A uniform circular disc has centre \(O\) and radius \(4 a\). The lines \(P Q\) and \(S T\) are perpendicular diameters of the disc. A circular hole of radius \(2 a\) is made in the disc, with the centre of the hole at the point \(R\) on \(O P\) where \(O R=2 a\), to form the lamina \(L\), shown shaded in Figure 2.
(a) Show that the distance of the centre of mass of \(L\) from \(P\) is \(\frac{14 a}{3}\).

The mass of \(L\) is \(m\) and a particle of mass \(k m\) is now fixed to \(L\) at the point \(P\). The system is now suspended from the point \(S\) and hangs freely in equilibrium. The diameter \(S T\) makes an angle \(\alpha\) with the downward vertical through \(S\), where \(\tan \alpha=\frac{5}{6}\).
(b) Find the value of \(k\).


\section*{Figure 3}

A small ball \(B\) of mass 0.25 kg is moving in a straight line with speed \(30 \mathrm{~m} \mathrm{~s}^{-1}\) on a smooth horizontal plane when it is given an impulse. The impulse has magnitude 12.5 Ns and is applied in a horizontal direction making an angle of \(\left(90^{\circ}+\alpha\right)\), where \(\tan \alpha=\frac{3}{4}\), with the initial direction of motion of the ball, as shown in Figure 3.
(i) Find the speed of \(B\) immediately after the impulse is applied.
(ii) Find the direction of motion of \(B\) immediately after the impulse is applied.
6. A car of mass 1200 kg pulls a trailer of mass 400 kg up a straight road which is inclined to the horizontal at an angle \(\alpha\), where \(\sin \alpha=\frac{1}{14}\). The trailer is attached to the car by a light inextensible towbar which is parallel to the road. The car's engine works at a constant rate of 60 kW . The non-gravitational resistances to motion are constant and of magnitude 1000 N on the car and 200 N on the trailer

At a given instant, the car is moving at \(10 \mathrm{~m} \mathrm{~s}^{-1}\). Find
(a) the acceleration of the car at this instant,
(b) the tension in the towbar at this instant.

The towbar breaks when the car is moving at \(12 \mathrm{~m} \mathrm{~s}^{-1}\).
(c) Find, using the work-energy principle, the further distance that the trailer travels before coming instantaneously to rest.

7


A small stone is projected from a point \(O\) at the top of a vertical cliff \(O A\). The point \(O\) is 52.5 m above the sea. The stone rises to a maximum height of 10 m above the level of \(O\) before hitting the sea at the point \(B\), where \(A B=50 \mathrm{~m}\), as shown in Figure 4. The stone is modelled as a particle moving freely under gravity.
(a) Show that the vertical component of the velocity of projection of the stone is \(14 \mathrm{~m} \mathrm{~s}^{-1}\).
(b) Find the speed of projection.
(c) Find the time after projection when the stone is moving parallel to \(O B\).

TOTAL FOR PAPER: 75 MARKS
END

6678

\section*{Edexcel GCE}

Mechanics M2

\section*{Advanced Level}

Friday 25 January 2013 - Afternoon
Time: \(\mathbf{1}\) hour 30 minutes
```

Materials required for examinatio
Mathematical Formulae (Pink)
Items included with question papers $\frac{\text { Item }}{\mathrm{Nil}}$

```
tor allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

\section*{Instructions to Candidates}
n the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of \(g\) is required, take \(g=9.8 \mathrm{~m} \mathrm{~s}^{-2}\).
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

\section*{Information for Candidate}

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75

\section*{Advice to Candidates}

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner Answers without working may not gain full credit.
1. Two uniform rods \(A B\) and \(B C\) are rigidly joined at \(B\) so that \(\angle A B C=90^{\circ}\). Rod \(A B\) has length 0.5 m and mass 2 kg . Rod \(B C\) has length 2 m and mass 3 kg . The centre of mass of the framework of the two rods is at \(G\).
(a) Find the distance of \(G\) from \(B C\)
(2)

The distance of \(G\) from \(A B\) is 0.6 m .
The framework is suspended from \(A\) and hangs freely in equilibrium
(b) Find the angle between \(A B\) and the downward vertical at \(A\).
2. A lorry of mass 1800 kg travels along a straight horizontal road. The lorry's engine is working at a constant rate of 30 kW . When the lorry's speed is \(20 \mathrm{~m} \mathrm{~s}^{-1}\), its acceleration is \(0.4 \mathrm{~m} \mathrm{~s}^{-2}\). The magnitude of the resistance to the motion of the lorry is \(R\) newtons.
(a) Find the value of \(R\).

The lorry now travels up a straight road which is inclined at an angle \(\alpha\) to the horizontal, where \(\sin \alpha=\frac{1}{12}\). The magnitude of the non-gravitational resistance to motion is \(R\) newtons. The lorry travels at a constant speed of \(20 \mathrm{~m} \mathrm{~s}^{-1}\).
(b) Find the new rate of working of the lorry's engine
3.
.


Figure 1
A ladder, of length 5 m and mass 18 kg , has one end \(A\) resting on rough horizontal ground and its other end \(B\) resting against a smooth vertical wall. The ladder lies in a vertical plane perpendicular to the wall and makes an angle \(\alpha\) with the horizontal ground, where \(\tan \alpha=\frac{4}{3}\), as shown in Figure 1. The coefficient of friction between the ladder and the ground is \(\mu\). A woman of mass 60 kg stands on the ladder at the point \(C\), where \(A C=3 \mathrm{~m}\). The ladder is on the point of slipping. The ladder is modelled as a uniform rod and the woman as a particle.

Find the value of \(\mu\).
4. At time \(t\) seconds the velocity of a particle \(P\) is \([(4 t-5) \mathbf{i}+3 \mathbf{j}] \mathrm{m} \mathrm{s}^{-1}\). When \(t=0\), the position vector of \(P\) is \((2 \mathbf{i}+5 \mathbf{j}) \mathrm{m}\), relative to a fixed origin \(O\).
(a) Find the value of \(t\) when the velocity of \(P\) is parallel to the vector \(\mathbf{j}\).
(b) Find an expression for the position vector of \(P\) at time \(t\) seconds.

A second particle \(Q\) moves with constant velocity \((-2 \mathbf{i}+c \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}\). When \(t=0\), the position vector of \(Q\) is \((11 \mathbf{i}+2 \mathbf{j}) \mathrm{m}\). The particles \(P\) and \(Q\) collide at the point with position vector \((d \mathbf{i}+14 \mathbf{j}) \mathrm{m}\).
(c) Find
(i) the value of \(c\),
(ii) the value of \(d\).
5. The point \(A\) lies on a rough plane inclined at an angle \(\theta\) to the horizontal, where \(\sin \theta=\frac{24}{25}\). A particle \(P\) is projected from \(A\), up a line of greatest slope of the plane, with speed \(U \mathrm{~m} \mathrm{~s}^{-1}\). The mass of \(P\) is 2 kg and the coefficient of friction between \(P\) and the plane is \(\frac{5}{12}\). The particle comes to instantaneous rest at the point \(B\) on the plane, where \(A B=1.5 \mathrm{~m}\). It then moves back down the plane to \(A\).
(a) Find the work done against friction as \(P\) moves from \(A\) to \(B\).
(b) Use the work-energy principle to find the value of \(U\).
(c) Find the speed of \(P\) when it returns to \(A\).
6.


\section*{Figure 2}

A ball is thrown from a point \(O\), which is 6 m above horizontal ground. The ball is projected with speed \(u \mathrm{~m} \mathrm{~s}^{-1}\) at an angle \(\theta\) above the horizontal. There is a thin vertical post which is 4 m high and 8 m horizontally away from the vertical through \(O\), as shown in Figure 2. The ball passes just above the top of the post 2 s after projection. The ball is modelled as a particle.
(a) Show that \(\tan \theta=2.2\)
(b) Find the value of \(u\).

The ball hits the ground \(T\) seconds after projection.
(c) Find the value of \(T\).

Immediately before the ball hits the ground the direction of motion of the ball makes an angle \(\alpha\) with the horizontal.
(d) Find \(\alpha\).
7. A particle \(A\) of mass \(m\) is moving with speed \(u\) on a smooth horizontal floor when it collides directly with another particle \(B\), of mass 3 m , which is at rest on the floor. The coefficient of restitution between the particles is \(e\). The direction of motion of \(A\) is reversed by the collision.
(a) Find, in terms of \(e\) and \(u\),
(i) the speed of \(A\) immediately after the collision,
(ii) the speed of \(B\) immediately after the collision.

After being struck by \(A\) the particle \(B\) collides directly with another particle \(C\), of mass \(4 m\), which is at rest on the floor. The coefficient of restitution between \(B\) and \(C\) is \(2 e\). Given that the direction of motion of \(B\) is reversed by this collision,
(b) find the range of possible values of \(e\)
(c) determine whether there will be a second collision between \(A\) and \(B\).

TOTAL FOR PAPER: 75 MARKS
END

\section*{6678/01R \\ Edexcel GCE}

\title{
Mechanics M2 (R) \\ Advanced/Advanced Subsidiary \\ Thursday 6 June 2013 - Morning \\ Time: \(\mathbf{1}\) hour \(\mathbf{3 0}\) minutes
}

\author{
Materials required for examination athematical Formulae (Pink) \\ Items included with question paper
}

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

This paper is strictly for students outside the UK.

\section*{Instructions to Candidates}

In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question. Whenever a numerical value of \(g\) is required, take \(g=9.8 \mathrm{~m} \mathrm{~s}^{-2}\).
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

\section*{Information for Candidate}

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (2).
There are 7 questions in this question paper. The total mark for this paper is 75
There are 28 pages in this question paper. Any blank pages are indicated.

\section*{Advice to Candidates}

You must ensure that your answers to parts of questions are clearly labelled
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

\section*{P42830A}
-
1. A caravan of mass 600 kg is towed by a car of mass 900 kg along a straight horizontal road. The towbar joining the car to the caravan is modelled as a light rod parallel to the road. The total resistance to motion of the car is modelled as having magnitude 300 N . The total resistance to motion of the caravan is modelled as having magnitude 150 N . At a given instant the car and the caravan are moving with speed \(20 \mathrm{~m} \mathrm{~s}^{-1}\) and acceleration \(0.2 \mathrm{~m} \mathrm{~s}^{-2}\).
(a) Find the power being developed by the car's engine at this instant.
(b) Find the tension in the towbar at this instant.
2. A ball of mass 0.2 kg is projected vertically upwards from a point \(O\) with speed \(20 \mathrm{~m} \mathrm{~s}^{-1}\). The non-gravitational resistance acting on the ball is modelled as a force of constant magnitude 1.24 N and the ball is modelled as a particle. Find, using the work-energy principle, the speed of the ball when it first reaches the point which is 8 m vertically above \(O\).
3. A particle \(P\) moves along a straight line in such a way that at time \(t\) seconds its velocity \(v \mathrm{~m} \mathrm{~s}^{-1}\) is given by
\[
v=\frac{1}{2} t^{2}-3 t+4
\]

Find
(a) the times when \(P\) is at rest,
(b) the total distance travelled by \(P\) between \(t=0\) and \(t=4\).
4. A rough circular cylinder of radius \(4 a\) is fixed to a rough horizontal plane with its axis horizontal. A uniform \(\operatorname{rod} A B\), of weight \(W\) and length \(6 a \sqrt{ } 3\), rests with its lower end \(A\) on the plane and a point \(C\) of the rod against the cylinder. The vertical plane through the rod is perpendicular to the axis of the cylinder. The rod is inclined at \(60^{\circ}\) to the horizontal, as shown in Figure 1.


Figure 1
(a) Show that \(A C=4 a \sqrt{ } 3\)
(2)

The coefficient of friction between the rod and the cylinder is \(\frac{\sqrt{ } 3}{3}\) and the coefficient of friction between the rod and the plane is \(\mu\). Given that friction is limiting at both \(A\) and \(C\),
(b) find the value of \(\mu\).
5. Two particles \(P\) and \(Q\), of masses \(2 m\) and \(m\) respectively, are on a smooth horizontal table. Particle \(Q\) is at rest and particle \(P\) collides directly with it when moving with speed \(u\). After the collision the total kinetic energy of the two particles is \(\frac{3}{4} m u^{2}\). Find
(a) the speed of \(Q\) immediately after the collision,
(b) the coefficient of restitution between the particles.
6.


A uniform triangular lamina \(A B C\) of mass \(M\) is such that \(A B=A C, B C=2 a\) and the distance of \(A\) from \(B C\) is \(h\). A line, parallel to \(B C\) and at a distance \(\frac{2 h}{3}\) from \(A\), cuts \(A B\) at \(D\) and cuts \(A C\) at \(E\), as shown in Figure 2.
It is given that the mass of the trapezium \(B C E D\) is \(\frac{5 M}{9}\).
(a) Show that the centre of mass of the trapezium \(B C E D\) is \(\frac{7 h}{45}\) from \(B C\).


\section*{Figure 3}

The portion \(A D E\) of the lamina is folded through \(180^{\circ}\) about \(D E\) to form the folded lamina shown in Figure 3.
(b) Find the distance of the centre of mass of the folded lamina from \(B C\).

The folded lamina is freely suspended from \(D\) and hangs in equilibrium. The angle between \(D E\) and the downward vertical is \(\alpha\).
(c) Find \(\tan \alpha\) in terms of \(a\) and \(h\).

\section*{6678/01 \\ Edexcel GCE}

\section*{Mechanics M2}

\section*{Advanced/Advanced Subsidiary}

Thursday 6 June 2013 - Morning
Time: 1 hour 30 minutes
```

Materials required for examination
Mathematical Formulae (Pink) (Pink)

```
\(\frac{\text { Items included with question papers }}{}\) \(\frac{\text { Item }}{\text { Nil }}\)

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

\section*{Instructions to Candidates}

In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper
Answer ALL the questions.
You must write your answer for each question in the space following the question
Whenever a numerical value of \(g\) is required, take \(g=9.8 \mathrm{~m} \mathrm{~s}^{-2}\).
When a calculator is used, the answer should be given to an appropriate degree of accuracy

\section*{Information for Candidates}
mulae and Statistical Tables' is provided

Full marks may be obtained for answers to ALL questions
The marks for the parts of questions are shown in round brackets, e.g. (2)
There are 7 questions in this question paper. The total mark for this paper is 75
There are 24 pages in this question paper. Any blank pages are indicated.

\section*{Advice to Candidates}

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examine Answers without working may not gain full credit
1. A particle \(P\) of mass 2 kg is moving with velocity \((\mathbf{i}-4 \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}\) when it receives an impulse of \((3 \mathbf{i}+6 \mathbf{j}) \mathrm{N} \mathrm{s}\)

Find the speed of \(P\) immediately after the impulse is applied.
2. A particle \(P\) of mass 3 kg moves from point \(A\) to point \(B\) up a line of greatest slope of a fixed rough plane. The plane is inclined at \(20^{\circ}\) to the horizontal. The coefficient of friction between \(P\) and the plane is 0.4

Given that \(A B=15 \mathrm{~m}\) and that the speed of \(P\) at \(A\) is \(20 \mathrm{~m} \mathrm{~s}^{-1}\), find
(a) the work done against friction as \(P\) moves from \(A\) to \(B\),
(b) the speed of \(P\) at \(B\).
3. A particle \(P\) moves on the \(x\)-axis. At time \(t\) seconds the velocity of \(P\) is \(v \mathrm{~m} \mathrm{~s}^{-1}\) in the direction of \(x\) increasing, where
\[
v=2 t^{2}-14 t+20, \quad t \geq 0
\]

Find
(a) the times when \(P\) is instantaneously at rest,
(b) the greatest speed of \(P\) in the interval \(0 \leq t \leq 4\)
(c) the total distance travelled by \(P\) in the interval \(0 \leq t \leq 4\).

\section*{P43997A}
4.


The uniform lamina \(A B C D E F\) is a regular hexagon with centre \(O\) and sides of length 2 m , as shown in Figure 1.


Figure 2
The triangles \(O A F\) and \(O E F\) are removed to form the uniform lamina \(O A B C D E\), shown in Figure 2.
(a) Find the distance of the centre of mass of \(O A B C D E\) from \(O\).

The lamina \(O A B C D E\) is freely suspended from \(E\) and hangs in equilibrium.
(b) Find the size of the angle between \(E O\) and the downward vertical.
5.


\section*{Figure 3}

A uniform \(\operatorname{rod} A B\), of mass \(m\) and length \(2 a\), is freely hinged to a fixed point \(A\). A particle of mass \(m\) is attached to the rod at \(B\). The rod is held in equilibrium at an angle \(\theta\) to the horizontal by a force of magnitude \(F\) acting at the point \(C\) on the rod, where \(A C=b\), as shown in Figure 3. The force at \(C\) acts at right angles to \(A B\) and in the vertical plane containing \(A B\).
(a) Show that \(F=\frac{3 a m g \cos \theta}{b}\).
(b) Find, in terms of \(a, b, g, m\) and \(\theta\),
(i) the horizontal component of the force acting on the \(\operatorname{rod}\) at \(A\),
(ii) the vertical component of the force acting on the rod at \(A\).

Given that the force acting on the \(\operatorname{rod}\) at \(A\) acts along the rod,
(c) find the value of \(\frac{a}{b}\).
6.


Figure 4
A ball is projected from a point \(A\) which is 8 m above horizontal ground as shown in Figure 4. The ball is projected with speed \(u \mathrm{~m} \mathrm{~s}^{-1}\) at an angle \(\theta^{\circ}\) above the horizontal. The ball moves freely under gravity and hits the ground at the point \(B\). The speed of the ball immediately before it hits the ground is \(2 u \mathrm{~m} \mathrm{~s}^{-1}\).
(a) By considering energy, find the value of \(u\).

The time taken for the ball to move from \(A\) to \(B\) is 2 seconds. Find
(b) the value of \(\theta\),
(c) the minimum speed of the ball on its path from \(A\) to \(B\).
7. Three particles \(P, Q\) and \(R\) lie at rest in a straight line on a smooth horizontal table with \(Q\) between \(P\) and \(R\). The particles \(P, Q\) and \(R\) have masses \(2 m, 3 m\) and \(4 m\) respectively. Particle \(P\) is projected towards \(Q\) with speed \(u\) and collides directly with it. The coefficient of restitution between each pair of particles is \(e\).
(a) Show that the speed of \(Q\) immediately after the collision with \(P\) is \(\frac{2}{5}(1+e) u\).

After the collision between \(P\) and \(Q\) there is a direct collision between \(Q\) and \(R\). Given that \(e=\frac{3}{4}\), find
(b) (i) the speed of \(Q\) after this collision,
(ii) the speed of \(R\) after this collision.

\section*{Immediately after the collision between \(Q\) and \(R\), the rate of increase of the distance between} \(P\) and \(R\) is \(V\).
(c) Find \(V\) in terms of \(u\).
(3)

\section*{TOTAL FOR PAPER: 75 MARKS}

END

\title{
WME02/01 \\ Pearson Edexcel \\ International Advanced Level
}

\section*{Mechanics M2}

\section*{Advanced/Advanced Subsidiary}

Friday 24 January 2014 - Afternoon
Time: 1 hour 30 minutes

\section*{Materials required for examination}

\section*{\(\frac{\text { Items included with question paper }}{\mathrm{Nil}}\)}

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, Qualifications. Calculators must not have the facility for symbolic algebra manipulation,
differentiation and integration, or have retrievable mathematical formulae stored in them. Instructions
- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B) Coloured pencils and highlighter pens must not be used
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled
- Answer the questions in the spaces provided
- there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of \(g\) is required, take \(g=9.8 \mathrm{~m} \mathrm{~s}^{-2}\), and give your answer to either two significant figures or three significant figures.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy

\section*{Information}
- The total mark for this paper is 75 .
- The marks for each question are shown in bracket
use this as a guide as to how much time to spend on each question

\section*{Advice}
- Read each question carefully before you start to answer it.
- Try to answer every question
- Check your answers if you have time at the end

\section*{P43069A}
1. A particle \(P\) of mass 2 kg is moving with velocity \((3 \mathbf{i}+4 \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}\) when it receives an impulse Immediately after the impulse is applied, \(P\) has velocity \((2 \mathbf{i}-3 \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}\).
(a) Find the magnitude of the impulse.
(b) Find the angle between the direction of the impulse and the direction of motion of \(P\) immediately before the impulse is applied.
2. A particle \(P\) moves on the \(x\)-axis. At time \(t\) seconds the velocity of \(P\) is \(v \mathrm{~m} \mathrm{~s}^{-1}\) in the direction of \(x\) increasing, where
\[
v=(t-2)(3 t-10), \quad t \geq 0
\]

When \(t=0, P\) is at the origin \(O\)
(a) Find the acceleration of \(P\) when \(t=3\).
(b) Find the total distance travelled by \(P\) in the first 3 seconds of its motion.
(c) Show that \(P\) never returns to \(O\).
3. A car has mass 550 kg . When the car travels along a straight horizontal road there is a constant resistance to the motion of magnitude \(R\) newtons, the engine of the car is working at a rate of \(P\) watts and the car maintains a constant speed of \(30 \mathrm{~m} \mathrm{~s}^{-1}\). When the car travels up a line of greatest slope of a hill which is inclined at \(\theta\) to the horizontal, where \(\sin \theta=\frac{1}{14}\), with the engine working at a rate of \(P\) watts, it maintains a constant speed of \(25 \mathrm{~m} \mathrm{~s}^{-1}\). The nongravitational resistance to motion when the car travels up the hill is a constant force of magnitude \(R\) newtons.
(a) (i) Find the value of \(R\).
(ii) Find the value of \(P\).
(b) Find the acceleration of the car when it travels along the straight horizontal road at \(20 \mathrm{~ms}^{-1}\) with the engine working at 50 kW .
4.


Figure 1
A uniform lamina \(A B C D\) is formed by removing the isosceles triangle \(A D C\) of height \(h\) metres, where \(h<2 \sqrt{ } 3\), from a uniform lamina \(A B C\) in the shape of an equilateral triangle of side 4 m , as shown in Figure 1. The centre of mass of \(A B C D\) is at \(D\).
(a) Show that \(h=\sqrt{3}\)

The weight of the lamina \(A B C D\) is \(W\) newtons. The lamina is freely suspended from \(A\). A horizontal force of magnitude \(F\) newtons is applied at \(B\) so that the lamina is in equilibrium with \(A B\) vertical. The horizontal force acts in the vertical plane containing the lamina.
(b) Find \(F\) in terms of \(W\).
5.


Figure 2
Figure 2 shows a uniform \(\operatorname{rod} A B\), of mass \(m\) and length \(2 a\), with the end \(B\) resting on rough horizontal ground. The rod is held in equilibrium at an angle \(\theta\) to the vertical by a light inextensible string. One end of the string is attached to the rod at the point \(C\), where \(A C=\frac{2}{3} a\). The other end of the string is attached to the point \(D\), which is vertically above \(B\), where \(B D=2 a\).
(a) By taking moments about \(D\), show that the magnitude of the frictional force acting on the \(\operatorname{rod}\) at \(B\) is \(\frac{1}{2} m g \sin \theta\).
(b) Find the magnitude of the normal reaction on the \(\operatorname{rod}\) at \(B\).

The rod is in limiting equilibrium when \(\tan \theta=\frac{4}{3}\)
(c) Find the coefficient of friction between the rod and the ground.
6. [In this question the unit vectors \(\mathbf{i}\) and \(\mathbf{j}\) are in a vertical plane, \(\mathbf{i}\) being horizontal and \(\mathbf{j}\) being vertically upwards.]


Figure 3
The point \(O\) is a fixed point on a horizontal plane. A ball is projected from \(O\) with velocity \((3 \mathbf{i}+v \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}, v>3\). The ball moves freely under gravity and passes through the point \(A\) before reaching its maximum height above the horizontal plane, as shown in Figure 3. The ball passes through \(A\) at time \(\frac{15}{49} \mathrm{~s}\) after projection. The initial kinetic energy of the ball is \(E\) joules. When the ball is at \(A\) it has kinetic energy \(\frac{1}{2} E\) joules.
(a) Find the value of \(v\).
7. Three particles \(A, B\) and \(C\), each of mass \(m\), lie at rest in a straight line \(L\) on a smooth horizontal surface, with \(B\) between \(A\) and \(C\). Particles \(A\) and \(B\) are projected directly towards each other with speeds \(5 u\) and \(4 u\) respectively. Particle \(C\) is projected directly away from \(B\) with speed \(3 u\). In the subsequent motion, \(A, B\) and \(C\) move along \(L\). Particles \(A\) and \(B\) collide directly. The coefficient of restitution between \(A\) and \(B\) is \(e\).
(a) (i) the speed of A immediately after the collision,
(ii) the speed of \(B\) immediately after the collision.

Given that the direction of motion of \(A\) is reversed in the collision between \(A\) and \(B\), and that there is no collision between \(B\) and \(C\),
(b) find the set of possible values of \(e\).

\title{
TOTAL FOR PAPER: 75 MARKS
}

END

At another point \(B\) on the path of the ball the kinetic energy is also \(\frac{1}{2} E\) joules.
The ball passes through \(B\) at time \(T\) seconds after projection.
(b) Find the value of \(T\).

\title{
WME02/01 \\ Pearson Edexcel \\ International Advanced Level
}

\section*{Mechanics M2 \\ Advanced/Advanced Subsidiary}

Monday 23 June 2014 - Morning
Time: \(\mathbf{1}\) hour \(\mathbf{3 0}\) minutes

\section*{Materials required for examination}

\section*{\(\frac{\text { Items included with question papers }}{\mathrm{Nil}}\)}

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, Qualifications. Calculators must not have the facility for symbolic algebra manipulation,
differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B) Coloured pencils and highlighter pens must not be used
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
- there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of \(g\) is required, take \(g=9.8 \mathrm{~m} \mathrm{~s}^{-2}\), and give your answer to either two significant figures or three significant figures.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy

\section*{Information}
- The total mark for this paper is 75 .
- The marks for each question are shown in brackets
use this as a guide as to how much time to spend on each question.

\section*{Advice}
- Read each question carefully before you start to answer it.
- Try to answer every question
- Check your answers if you have time at the end

P44520A
1. A particle \(P\) moves on the \(x\)-axis. The acceleration of \(P\), in the positive \(x\) direction at time \(t\) seconds, is \((2 t-3) \mathrm{m} \mathrm{s}^{-2}\). The velocity of \(P\), in the positive \(x\) direction at time \(t\) seconds, is \(v \mathrm{~m} \mathrm{~s}^{-1}\). When \(t=0, v=2\).
(a) Find \(v\) in terms of \(t\).

The particle is instantaneously at rest at times \(t_{1}\) seconds and \(t_{2}\) seconds, where \(t_{1}<t_{2}\).
(b) Find the values \(t_{1}\) and \(t_{2}\)
(c) Find the distance travelled by \(P\) between \(t=t_{1}\) and \(t=t_{2}\).
2. A trailer of mass 250 kg is towed by a car of mass 1000 kg . The car and the trailer are travelling down a straight road inclined at an angle \(\theta\) to the horizontal, where \(\sin \theta=\frac{1}{20}\).

The resistance to motion of the car is modelled as a single force of magnitude 300 N acting parallel to the road. The resistance to motion of the trailer is modelled as a single force of magnitude 100 N acting parallel to the road. The towbar joining the car to the trailer is modelled as a light rod which is parallel to the direction of motion. At a given instant the car and the trailer are moving down the road with speed \(25 \mathrm{~m} \mathrm{~s}^{-1}\) and acceleration \(0.2 \mathrm{~m} \mathrm{~s}^{-2}\).
(a) Find the power being developed by the car's engine at this instant.
(b) Find the tension in the towbar at this instant.


\section*{Figure 1}

A uniform \(\operatorname{rod} A B\) of weight \(W\) is freely hinged at end \(A\) to a vertical wall. The rod is supported in equilibrium at an angle of \(60^{\circ}\) to the wall by a light rigid strut \(C D\). The strut is freely hinged to the rod at the point \(D\) and to the wall at the point \(C\), which is vertically below \(A\), as shown in Figure 1. The rod and the strut lie in the same vertical plane, which is perpendicular to the wall. The length of the rod is \(4 a\) and \(A C=A D=2.5 a\).
(a) Show that the magnitude of the thrust in the strut is \(\frac{4 \sqrt{ } 3}{5} W\).
(b) Find the magnitude of the force acting on the rod at \(A\).
4.


Figure 2
The uniform square lamina \(A B C D\) shown in Figure 2 has sides of length \(4 a\). The points \(E\) and \(F\), on \(D A\) and \(D C\) respectively, are both at a distance \(3 a\) from \(D\).

The portion \(D E F\) of the lamina is folded through \(180^{\circ}\) about \(E F\) to form the folded lamina \(A B C F E\) shown in Figure 3 below.


Figure 3
(a) Show that the distance from \(A B\) of the centre of mass of the folded lamina is \(\frac{55}{32} a\).

The folded lamina is freely suspended from \(E\) and hangs in equilibrium.
(b) Find the size of the angle between \(E D\) and the downward vertical
5. A particle of mass 0.5 kg is moving on a smooth horizontal surface with velocity \(12 \mathbf{i} \mathrm{~m} \mathrm{~s}^{-1}\) when it receives an impulse \(K(\mathbf{i}+\mathbf{j}) \mathrm{N} \mathrm{s}\), where \(K\) is a positive constant. Immediately after receiving the impulse the particle is moving with speed \(15 \mathrm{~m} \mathrm{~s}^{-1}\) in a direction which makes an acute angle \(\theta\) with the vector \(\mathbf{i}\)

Find
(i) the value of \(K\),
(ii) the size of angle \(\theta\)
6. Three particles \(P, Q\) and \(R\) have masses \(3 \mathrm{~m}, \mathrm{~km}\) and 7.5 m respectively. The three particles lie at rest in a straight line on a smooth horizontal table with \(Q\) between \(P\) and \(R\). Particle \(P\) is projected towards \(Q\) with speed \(u\) and collides directly with \(Q\). The coefficient of restitution between \(P\) and \(Q\) is \(\frac{1}{9}\).
(a) Show that the speed of \(Q\) immediately after the collision is \(\frac{10 u}{3(3+k)}\).
(b) Find the range of values of \(k\) for which the direction of motion of \(P\) is reversed as a result of the collision.

Following the collision between \(P\) and \(Q\) there is a collision between \(Q\) and \(R\). Given that \(k=7\) and that \(Q\) is brought to rest by the collision with \(R\)
(c) find the total kinetic energy lost in the collision between \(Q\) and \(R\).
7. A particle \(P\) is projected from a fixed point \(A\) with speed \(4 \mathrm{~m} \mathrm{~s}^{-1}\) at an angle \(\alpha\) above the horizontal and moves freely under gravity. When \(P\) passes through the point \(B\) on its path, it has speed \(7 \mathrm{~m} \mathrm{~s}^{-1}\).
(a) By considering energy, find the vertical distance between \(A\) and \(B\).

The minimum speed of \(P\) on its path from \(A\) to \(B\) is \(2.5 \mathrm{~m} \mathrm{~s}^{-1}\).
(b) Find the size of angle \(\alpha\).
(c) Find the horizontal distance between \(A\) and \(B\).

\section*{TOTAL FOR PAPER: 75 MARKS}

END

\section*{6678/01R \\ Edexcel GCE}

\title{
Mechanics M2 (R) \\ Advanced/Advanced Subsidiary \\ Monday 23 June 2014 - Morning \\ \\ Time: \(\mathbf{1}\) hour 30 minutes
} \\ \\ Time: \(\mathbf{1}\) hour 30 minutes
}
```

Materials required for examination
Thematical Formulae (Pink)

```

\section*{ems included with question paper Nil}

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have
retrievable mathematical formulae stored in them.
This paper is strictly for students outside the UK.

\section*{Instructions to Candidate}

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678R), your surname, initials and signature. Whenever a numerical value of \(g\) is required, take \(g=9.8 \mathrm{~m} \mathrm{~s}^{-2}\).
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

\section*{Information for Candidates}

A booklet 'Mathematical Formulae and Statistical Tables' is provided
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (2).
There are 7 questions in this question paper. The total mark for this paper is 75

\section*{Advice to Candidates}

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner
Answers without working may not gain full credit.
1. A van of mass 600 kg is moving up a straight road inclined at an angle \(\theta\) to the horizontal, where \(\sin \theta=\frac{1}{16}\). The resistance to motion of the van from non-gravitational forces has constant magnitude \(R\) newtons. When the van is moving at a constant speed of \(20 \mathrm{~m} \mathrm{~s}^{-1}\), the van's engine is working at a constant rate of 25 kW .
(a) Find the value of \(R\)

The power developed by the van's engine is now increased to 30 kW . The resistance to motion from non-gravitational forces is unchanged. At the instant when the van is moving up the road at \(20 \mathrm{~m} \mathrm{~s}^{-1}\), the acceleration of the van is \(a \mathrm{~m} \mathrm{~s}^{-2}\).
(b) Find the value of \(a\)
2. A ball of mass 0.4 kg is moving in a horizontal plane when it is struck by a bat. The bat exerts an impulse \((-5 \mathbf{i}+3 \mathbf{j}) \mathrm{Ns}\) on the ball. Immediately after receiving the impulse the ball has velocity \((12 \mathbf{i}+15 \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}\).

Find
(a) the speed of the ball immediately before the impact,
(b) the size of the angle through which the direction of motion of the ball is deflected by the impact.
3.


\section*{Figure 1}

A non-uniform rod, \(A B\), of mass \(m\) and length \(2 l\), rests in equilibrium with one end \(A\) on a rough horizontal floor and the other end \(B\) against a rough vertical wall. The rod is in a vertical plane perpendicular to the wall and makes an angle of \(60^{\circ}\) with the floor as shown in Figure 1. The coefficient of friction between the rod and the floor is \(\frac{1}{4}\) and the coefficient of friction between the rod and the wall is \(\frac{2}{3}\). The rod is on the point of slipping at both ends.
(a) Find the magnitude of the vertical component of the force exerted on the rod by the floor.

The centre of mass of the rod is at \(G\).
(b) Find the distance \(A G\).
4.


\section*{Figure 2}

Figure 2 shows a lamina \(L\). It is formed by removing a square \(P Q R S\) from a uniform triangle \(A B C\). The triangle \(A B C\) is isosceles with \(A C=B C\) and \(A B=12 \mathrm{~cm}\). The midpoint of \(A B\) is \(D\) and \(D C=8 \mathrm{~cm}\). The vertices \(P\) and \(Q\) of the square lie on \(A B\) and \(P Q=4 \mathrm{~cm}\). The centre of the square is \(O\). The centre of mass of \(L\) is at \(G\).
(a) Find the distance of \(G\) from \(A B\).

When \(L\) is freely suspended from \(A\) and hangs in equilibrium, the line \(A B\) is inclined at \(25^{\circ}\) to the vertical.
(b) Find the distance of \(O\) from \(D C\).


\section*{Figure 3}

A particle \(P\) of mass 2 kg is released from rest at a point \(A\) on a rough inclined plane and slides down a line of greatest slope. The plane is inclined at \(30^{\circ}\) to the horizontal. The point \(B\) is 5 m from \(A\) on the line of greatest slope through \(A\), as shown in Figure 3 .
(a) Find the potential energy lost by \(P\) as it moves from \(A\) to \(B\).

The speed of \(P\) as it reaches \(B\) is \(4 \mathrm{~m} \mathrm{~s}^{-1}\).
(b) (i) Use the work-energy principle to find the magnitude of the constant frictional force acting on \(P\) as it moves from \(A\) to \(B\).
(ii) Find the coefficient of friction between \(P\) and the plane.

The particle \(P\) is now placed at \(A\) and projected down the plane towards \(B\) with speed \(3 \mathrm{~m} \mathrm{~s}^{-1}\). Given that the frictional force remains constant,
(c) find the speed of \(P\) as it reaches \(B\)
6.


\section*{Figure 4}

A particle \(P\) is projected from a point \(A\) with speed \(25 \mathrm{~m} \mathrm{~s}^{-1}\) at an angle of elevation \(\alpha\), where \(\sin \alpha=\frac{4}{5}\). The point \(A\) is 10 m vertically above the point \(O\) which is on horizontal ground, as shown in Figure 4. The particle \(P\) moves freely under gravity and reaches the ground at the point \(B\).

Calculate
(a) the greatest height above the ground of \(P\), as it moves from \(A\) to \(B\),
(b) the distance \(O B\).

The point \(C\) lies on the path of \(P\). The direction of motion of \(P\) at \(C\) is perpendicular to the direction of motion of \(P\) at \(A\).
(c) Find the time taken by \(P\) to move from \(A\) to \(C\).
7. A particle \(P\) of mass \(2 m\) is moving in a straight line with speed \(3 u\) on a smooth horizontal table. A second particle \(Q\) of mass \(3 m\) is moving in the opposite direction to \(P\) along the same straight line with speed \(u\). The particle \(P\) collides directly with \(Q\). The direction of motion of \(P\) is reversed by the collision. The coefficient of restitution between \(P\) and \(Q\) is \(e\)
(a) Show that the speed of \(Q\) immediately after the collision is \(\frac{u}{5}(8 e+3)\).
(b) Find the range of possible values of \(e\).

The total kinetic energy of the particles before the collision is \(T\). The total kinetic energy of the particles after the collision is \(k T\). Given that \(e=\frac{1}{2}\),
(c) find the value of \(k\)

\section*{6678/01 \\ Edexcel GCE}

\section*{Mechanics M2}

Advanced/Advanced Subsidiary
Monday 23 June 2014 - Morning
Time: \(\mathbf{1}\) hour \(\mathbf{3 0}\) minutes

\section*{Materials required for examination athematical Formulae (Pink) \\ tems included with question paper \(\xrightarrow{\text { Nil }}\)}

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

\section*{nstructions to Candidates}

In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper. Answer ALL the questions.
You must write your answer for each question in the space following the question. Whenever a numerical value of \(g\) is required, take \(g=9.8 \mathrm{~m} \mathrm{~s}^{-2}\).
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

\section*{Information for Candidate}

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (2).
There are 8 questions in this question paper. The total mark for this paper is 75
There are 28 pages in this question paper. Any blank pages are indicated.

\section*{Advice to Candidates}

You must ensure that your answers to parts of questions are clearly labelled You must show sufficient working to make your methods clear to the Examiner. Answers without working may not gain full credit.

\section*{43171A}
\(\qquad\)
1. Three particles of mass \(3 \mathrm{~m}, 2 \mathrm{~m}\) and km are placed at the points whose coordinates are \((1,5)\), \((6,4)\) and \((a, 1)\) respectively. The centre of mass of the three particles is at the point with coordinates \((3,3)\).

Find
(a) the value of \(k\),
(b) the value of \(a\).
2. At time \(t\) seconds, where \(t \geq 0\), a particle \(P\) is moving on a horizontal plane with acceleration \(\left[\left(3 t^{2}-4 t\right) \mathbf{i}+(6 t-5) \mathbf{j}\right] \mathrm{m} \mathrm{s}^{-2}\).

When \(t=3\) the velocity of \(P\) is \((11 \mathbf{i}+10 \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}\).
Find
(a) the velocity of \(P\) at time \(t\) seconds,
(b) the speed of \(P\) when it is moving parallel to the vector \(\mathbf{i}\).
3.


Figure 1
The uniform lamina \(A B C D E F\), shown shaded in Figure 1, is symmetrical about the line through \(B\) and \(E\). It is formed by removing the isosceles triangle \(F E D\), of height \(6 a\) and base \(8 a\), from the isosceles triangle \(A B C\) of height \(9 a\) and base \(12 a\).
(a) Find, in terms of \(a\), the distance of the centre of mass of the lamina from \(A C\).

The lamina is freely suspended from \(A\) and hangs in equilibrium.
(b) Find, to the nearest degree, the size of the angle between \(A B\) and the downward vertical.
4. A truck of mass 1800 kg is towing a trailer of mass 800 kg up a straight road which is inclined to the horizontal at an angle \(\alpha\), where \(\sin \alpha=\frac{1}{20}\). The truck is connected to the trailer by a light inextensible rope which is parallel to the direction of motion of the truck. The resistances to motion of the truck and the trailer from non-gravitational forces are modelled as constant forces of magnitudes 300 N and 200 N respectively. The truck is moving at constant speed \(v \mathrm{~m} \mathrm{~s}^{-1}\) and the engine of the truck is working at a rate of 40 kW
(a) Find the value of \(v\)
(5)

As the truck is moving up the road the rope breaks.
(b) Find the acceleration of the truck immediately after the rope breaks.
5. A particle of mass \(m \mathrm{~kg}\) lies on a smooth horizontal surface. Initially the particle is at rest at a point \(O\) midway between a pair of fixed parallel vertical walls. The walls are 2 m apart. At time \(t=0\) the particle is projected from \(O\) with speed \(u \mathrm{~m} \mathrm{~s}^{-1}\) in a direction perpendicular to the walls. The coefficient of restitution between the particle and each wall is \(\frac{2}{3}\). The magnitude of the impulse on the particle due to the first impact with a wall is \(\lambda m u \mathrm{~N} \mathrm{~s}\)
(a) Find the value of \(\lambda\).

The particle returns to \(O\), having bounced off each wall once, at time \(t=3\) seconds.
(b) Find the value of \(u\).
6.


Figure 2
A small ball is projected with speed \(14 \mathrm{~m} \mathrm{~s}^{-1}\) from a point \(A\) on horizontal ground. The angle of projection is \(\alpha\) above the horizontal. A horizontal platform is at height \(h\) metres above the ground. The ball moves freely under gravity until it hits the platform at the point \(B\), as shown in Figure 2. The speed of the ball immediately before it hits the platform at \(B\) is \(10 \mathrm{~m} \mathrm{~s}^{-1}\).
(a) Find the value of \(h\).

Given that \(\sin \alpha=0.85\),
(b) find the horizontal distance from \(A\) to \(B\).

\section*{Figure 3}

A uniform \(\operatorname{rod} A B\) of weight \(W\) has its end \(A\) freely hinged to a point on a fixed vertical wall. The rod is held in equilibrium, at angle \(\theta\) to the horizontal, by a force of magnitude \(P\). Th force acts perpendicular to the rod at \(B\) and in the same vertical plane as the rod, as shown in Figure 3. The rod is in a vertical plane perpendicular to the wall. The magnitude of the vertical component of the force exerted on the rod by the wall at \(A\) is \(Y\).
(a) Show that \(Y=\frac{W}{2}\left(2-\cos ^{2} \theta\right)\).

Given that \(\theta=45^{\circ}\)
(b) find the magnitude of the force exerted on the rod by the wall at \(A\), giving your answer in terms of \(W\).
8. The points \(A\) and \(B\) are 10 m apart on a line of greatest slope of a fixed rough inclined plane, with \(A\) above \(B\). The plane is inclined at \(25^{\circ}\) to the horizontal. A particle \(P\) of mass 5 kg i released from rest at \(A\) and slides down the slope. As \(P\) passes \(B\), it is moving with speed \(7 \mathrm{~m} \mathrm{~s}^{-1}\).
(a) Find, using the work-energy principle, the work done against friction as \(P\) moves from \(A\) to \(B\).
(b) Find the coefficient of friction between the particle and the plane.

\title{
TOTAL FOR PAPER: 75 MARKS
}

END```

